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★**Differential geometry and Lie groups for physicists.**
Cambridge University Press, Cambridge, 2006. *xvi*+697 *pp*. $75.00.
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As the title indicates this textbook is about modern differential geometry meant for (theoretical) physicists, and it does a very good job of motivating every new concept and definition based on knowledge a graduate or advanced undergraduate student would have from courses in mechanics and, in some cases, quantum mechanics. The mathematics background required is limited to calculus, linear algebra and some real analysis. The author writes in an informal style, forcing the serious readers to verify almost every result by their own calculation or simple proof. Most of the exercises are followed by hints which may be just that, but in some cases amount to almost complete proofs. There are no formal statements and proofs of theorems, but some are informally discussed and always well motivated and illustrated by examples that demonstrate the cases when they apply or do not apply.

The contents of this book covers a lot (if not most) of what a theoretical physicist might wish to know about differential geometry and Lie groups. Particularly useful may be that the modern formalism is always related to the classical one with tensor indices still mostly used in the physics literature. (Often a good argument is made to demonstrate the superiority of the former.) What is not included are all topological aspects except the basic de Rham cohomology. Also, precise statements of global results and many theorems (like Whitney, Darboux, Frobenius, etc.) are only briefly and informally referred to. What might be considered a disadvantage by some serious students is the complete lack of any precise references to such proofs. The brief bibliography at the end of the book appears to be a somewhat arbitrary collection of related textbooks.

The following is a more detailed list of the contents. The first nine chapters introduce manifolds, mappings, tensor fields, Lie derivatives, the exterior algebra and calculus, the definition of integration, Stokes' theorem with applications, the Poincaré lemma and de Rham cohomology. The following four chapters discuss Lie groups, Lie algebras, basic concepts of Lie group representation theory, actions of Lie groups on manifolds and homogeneous spaces. Chapter 14 is on symplectic geometry and Hamiltonian mechanics with an emphasis on symmetries including a discussion of the momentum mapping and symplectic reduction. This topic is taken up again in Chapter 18 on Lagrange's and Hamilton's equations. Chapter 15 includes a relatively brief treatment of linear connections, Riemannian metrics, geodesics, torsion and curvature. It is followed by a chapter on applications to the classical field theory of physics. In Chapters 17, 19 and 20 the geometry of fibre bundles is introduced, carefully motivated by the tangent and cotangent bundles, followed by linear connections on the frame bundle and finally connections on arbitrary principal bundles. The latter are applied in Chapter 21, dealing with classical gauge theory, and in the final chapter, which is on spinor fields. This last chapter contains a quite detailed discussion of Clifford algebras in arbitrary dimensions and the associated $\mathrm{Spin}(p, q)$ groups, spinor bundles, and Dirac operators.

Reviewed by *Hans-Peter Künzle*