Newton's pail in Einstein's lift

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The classic Newton's pail experiment performed in a freely falling lift is discussed.

I. INTRODUCTION

Newton proposed his celebrated pail experiment in order to support the concept of absolute space (and distinguished-inertial-reference frames), especially in connection with an objection of his contemporary Leibnitz, who protested against such a concept on philosophical grounds. Let us recall the experiment in a few words:

"He filled a pail with water and suspended it from a twisted rope. In unwinding itself the rope set the pail into rotary motion, and the rotation of the pail continued for a while until it came to rest. The water in the pail was at rest in the first stage of the rotation of the pail and had a level surface. The fact that the pail was moving relative to it did not affect it. In the second phase of the rotation of the pail, the friction between the fluid and the wall forced the fluid to participate in the motion. Water and pail then moved as one body, and, according to Newton, the surface of the water had the form of a paraboloid of revolution due to the centrifugal force on the water. In the third stage, the pail had already come to rest, but the water was still rotating. In a certain sense the situation was similar to the first stage: water and pail were in the same relative motion. But now the surface of the water was parabolic. This showed that not the relative motion of water and pail were decisive for the phenomenon of depression of the water surface, but the rotation of the body of water relative to absolute space and the consequent centrifugal force. The Leibnitz objection was thus overruled by experiment."[1]

The other "experimental setup" mentioned in the title, Einstein's lift,[2] is certainly not less classic. It enables one to transform away ("switch off") locally a gravitational force.

An interesting question now arises: what happens if the rope keeping the pail filled with a uniformly rotating water is suddenly cut (Fig. 1), or in other words, what is the behavior of *Newton's pail in Einstein's lift?*

II. WHAT HAPPENS

One should first of all realize clearly the reason for the paraboloid of revolution shape of the surface of the water (*before* the rope is cut). There are two forces governing the behavior of the water (when viewed from the co-moving = rotating frame of reference; for the explanation within the laboratory frame by solving Euler equations for ideal liquid see Ref. 3). Gravitation forces the water to sit as low as possible while due to the centrifugal force the water tries to occur as far as possible from the axis of rotation, which means, however, to climb up (for the usual shape of the pail) just against the gravitational force. The resulting form of the surface is merely a "compromise agreement" between these two contrary tendencies.

When the rope is cut, the gravitational force is disqualified from taking part in the competition and the only competitor available—the centrifugal force—wins with glorification. The water climbs up with no resistance until it finally *pours out* of the pail.

III. FOUR VERSIONS OF PAIL EXPERIMENT

Both forces mentioned in Sec. II are under control—we can switch off and on separately each of them (gravitational—"to cut or not to cut," centrifugal—"to rotate or not to rotate"). Thus, four different situations can occur from this point of view (the first and the third stage from Sec. I plus both of them after the rope is cut). They are summarized in the following table:

No.	Gravitational force	Centrifugal force	Result
1	yes	yes	parabolic surface
2	yes	no	flat surface
3	no	yes	water pours out
4	no	no	flat surface

(In No. 4 it is assumed that the surface was flat before the rope is cut.)

IV. SIMPLIFIED (KITCHEN) REALIZATION OF EXPERIMENT

A "standard" realization using a pail, rope, scissors, etc. is straightforward but a bit cumbersome to perform (e.g.,

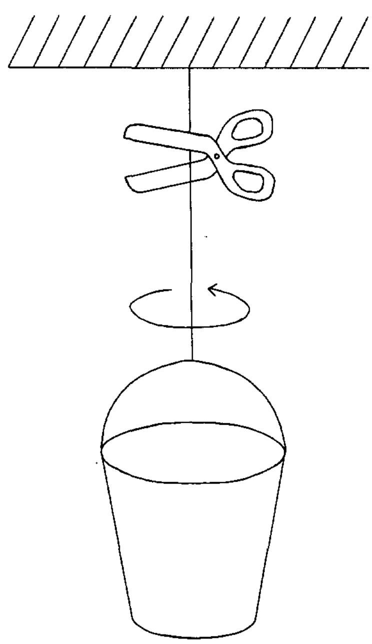

Fig. 1. What happens if the rope is suddenly cut?

in the lecture room). One can demonstrate, however, all essential features of the experiment equipped with nothing more than a glass of water and a teaspoon.

First, one has to gain some skill in performing No. 4 experiment (to drop the glass half-filled with water and catch it again keeping the water inside) and then verify that *no skill* can help to confine the water to the glass if the water is *stirred* previously (experiment No. 3 is necessarily "wet").

V. DISCUSSION

As mentioned in Sec. I the moral from the Newton's pail experiment (No. 1 compared with No. 2) is that the relative motion of water and "absolute space" rather than water and pail is decisive for the result. The curvature of the surface serves, in fact, as an indicator of the measuring instrument which measures quantitatively the relative motion of the water and absolute space. From the table in Sec. III. we see, however, that it is possible to produce even more pronounced deviation of the indicator (or better a damaging of the apparatus because of too strong signal) performing experiment No. 3. Although the relative motion of water and pail can be still the same, the result differs from that of No. 2 much more dramatically (water outside vs water inside the pail) in comparison with No. 1 (surface curved vs surface flat). Thus, No. 3 experiment can be viewed as in a sense strengthened version of the original Newton's experiment (the strenghtening is caused by the fact that the rest frame of the water just after the cut in No. 3 is in more complicated relation to the "absolute space" than in No. 1: "linear" acceleration is added to rotation).

[1]R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965), p. 2.

[2]A. Einstein and L. Infeld, *Die Evolution der Physik* (Paul Zsolnay Verlag, Wien, 1950), p. 235.

[3]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959), p. 24.

Numerical integration of Newton's equations including velocity-dependent forces

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Numerical integration routines designed for introductory physics courses, such as the last-point approximation and the second Taylor approximation, are incompatible with velocity-dependent forces. A general purpose routine which handles resistive, Coriolis, and magnetic forces, as well as conservative forces, is obtained by combining the fundamental Euler method with Richardson extrapolation. Further, this Euler–Richardson method is almost as efficient as the last-point approximation and the second Taylor approximation for simple central force problems and is more efficient for difficult problems, such as Earth–Moon orbits.

I. INTRODUCTION

The use of numerical integration techniques in introductory physics courses supposes that the methods satisfy two requirements: (a) they can be easily implemented and understood by the students and (b) they can solve the problems. To a large extent, these are antagonistic features. The simplest method is the fundamental Euler method (denoted by FEM in this paper), but this is very limited in its ability to handle interesting problems. On the other hand, high quality methods such as the fifth-order Dormand–Prince method[1] (DP5) or the Bulirsch–Stoer technique[2] (BST), which are the culmination of years of development, are very sophisticated and not easily explained. As a result, a number of slightly refined methods have been developed for student use, typically based on differential equations that are purely second order. These include the midpoint approximation (MPA), the half-step approximation (HSA), the last-point approximation (LPA),[3] and the second Taylor approximation (STA).[4] The second Taylor approximation has the additional advantage of being able to use variable step sizes.[4,5] Of these, the LPA and the STA have received most attention.

The particular applications in mind during the development of the refined methods have all been cases of Newton's Second Law with conservative forces; $d^2x/dt^2 = F(x)/m$, or the equivalent in two dimensions. But it has also been assumed that the real purpose is to examine variations of these forces, including the addition of resistive forces.[3,4] Unfortunately, all these slightly refined methods are incompatible with velocity-dependent forces, a feature that has not been recognized (except in one brief *note added in proof*)[4] and never discussed in detail.

The numerical integration of dynamics equations requires a large number of acceleration calculations, many of which are used to refine earlier estimates of position and velocity. If the acceleration can be determined using only position information, without regard for the velocity, this refinement process is simplified and it is possible to obtain