MATHEMATICAL PHYSICS 2
Syllabus of a graduate course at FMFI, Comenius University in Bratislava
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5. EXTERIOR ALGEBRA

5.1. Motivation - volumes of parallelepipeds
5.2. p-forms and exterior product
5.3. Exterior algebra ΛL^*
5.4. Interior product i_v
5.5. Orientation in L
5.6. Determinant and generalized Kronecker symbols
5.7. The metric volume form
5.8. Hodge (duality) operator $*$

6. DIFFERENTIAL CALCULUS OF FORMS

6.1. Forms on a manifold
6.2. Exterior derivative
6.3. Orientability, Hodge operator and volume form on M

7. INTEGRAL CALCULUS OF FORMS

7.1. Quantities under the integral sign regarded as differential forms
7.2. Euclidean simplices and chains
7.3. Simplices and chains on a manifold
7.4. Integral of a form over a chain on a manifold
7.5. Stokes' theorem
7.6. Integral over a domain on an orientable manifold
7.7. Integral over a domain on an orientable Riemannian manifold
7.8. Integral and maps of manifolds

8. PARTICULAR CASES AND APPLICATIONS OF STOKES' THEOREM

8.1. Elementary situations
8.2. Divergence of a vector field and Gauss' theorem
8.3. Codifferential and Laplace-deRham operator
8.4. Green identities
8.5. Vector analysis in E^3
8.6. Functions of complex variables

9. POINCARÉ LEMMA AND COHOMOLOGIES

9.1. Simple examples of closed non-exact forms
9.2. Construction of a potential on contractible manifolds
9.3. Cohomologies and deRham complex

14. HAMILTONIAN MECHANICS AND SYMPLECTIC MANIFOLDS

14.1. Poisson and symplectic structure on a manifold
14.2. Darboux theorem, canonical transformations and symplectomorphisms
14.3. Poincaré-Cartan integral invariants and Liouville's theorem

15. PARALLEL TRANSPORT AND LINEAR CONNECTION ON M

15.1. Acceleration and parallel transport
15.2. Parallel transport and covariant derivative
15.3. Compatibility with metric, RLC connection
15.4. Geodesics

15.5. The curvature tensor
15.6. Connection forms and Cartan structure equations

16. FIELD THEORY AND THE LANGUAGE OF FORMS

16.1. Differential forms in the Minkowski space $E^{1,3}$
16.2. Maxwell equations in terms of differential forms
16.3. Gauge transformations, action integral

11. DIFFERENTIAL GEOMETRY ON LIE GROUPS

11.1. Left-invariant tensor fields on a Lie group
11.2. Lie algebra $\mathcal{G}$ of a group G
11.3. Invariant integral on G

(Numbering of sections follows the textbook 4,5.)

REFERENCES

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