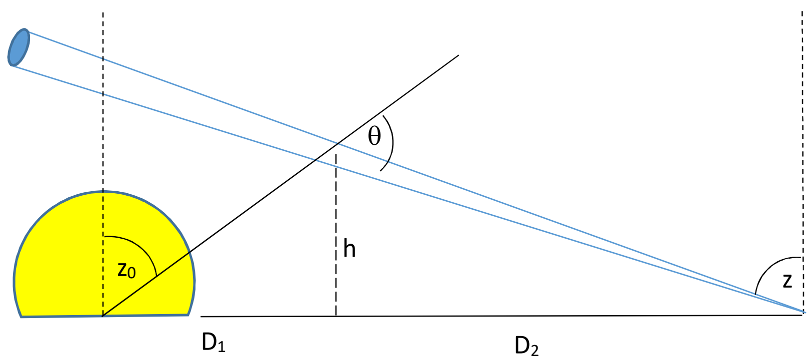
Ferov student vyvinul sw na identifikaciu spectra zdrojov svetla v meste na zaklade spektralnych merani svetelneho dómu nad mestom. Dolezite je vediet ako sa zhruba transformuje spektrum zdroja do spektra oblohy nad mestom. Jednoduchy model je dole:



Zjednoduseny model na meridiane svetelneho zdroja: $\theta = \pi - z - z_0,\ z_0 \in \left[-z, \frac{\pi}{2}\right]$

Merany signal na zenitovom uhle z:

$$I(z) \propto \frac{Q_0}{\cos z} \int_0^{\infty} \left(\frac{\cos z_0}{h}\right)^2 T(h, z, z_0)\, k_{sca}(h, \theta)\, dh \qquad \text{(Eq)}$$

$$\tan z_0 = \frac{D_1}{h} \qquad \tan z = \frac{D_2}{h} \qquad \mathrm{h} = \frac{D_1 + D_2}{\tan z + \tan z_0} = \frac{D}{\tan z + \tan z_0} \qquad \mathrm{dh} = -\frac{h^2}{D} \frac{dz_0}{\cos^2 z_0}$$

Preto

$$I(z) \propto \frac{Q_0}{\cos z} \frac{1}{D} \int_{-z}^{\pi/2} k_{sca}(\theta)\, dz_0 \qquad \text{(Eq)}$$

Predpokladam, ze najvyznamnejsia cast signal pochadza zo spodnej vrstvy atmosfery, kde je kvoli aktivnemu turbulentnemu premiesavaniu koncentracia znecistujucich primesi malo zavisla na vyske nad povrchom zeme a preto:

$$k_{sca}(\theta) = \frac{1}{4\pi} [k_R P_R(\theta) + k_A P_A(\theta)] \qquad \text{(Eq)}$$

kde k_R je koeficient rozptylu pre Rayleigho rozptyl a P_R je zodpovedajuca fazova funkcia rozptylu. Obdobne k_A a P_A su koeficient rozptylu a fazova funkcia pre aerosolovu komponentu atmosfery.

$$k_{sca}(\theta) = \frac{k_R}{4\pi} \frac{3}{4} (1 + \cos^2 \theta) + \frac{k_A}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}} \qquad \text{(Eq)}$$

Rayleigho zlozka intenzity:

$$I_R(z) \propto \frac{Q_0}{\cos z} \frac{1}{D} \frac{3k_R}{16\pi} \int_{-z}^{\pi/2} [1 + \cos^2(\pi - z - z_0)]\, dz_0 = \frac{Q_0}{\cos z} \frac{1}{D} \frac{3k_R}{16\pi} \left(\frac{\pi}{4} + \frac{z}{2} + \frac{\sin 2z}{4}\right) \qquad \text{(Eq)}$$

Aerosolova zlozka intenzity:

$$I_A(z) \propto \frac{Q_0}{\cos z} \frac{(1-g^2)}{D} \frac{k_A}{4\pi} \int_{-z}^{\pi/2} \frac{dz_0}{[1+g^2-2g\cos(\pi-z-z_0)]^{3/2}} \quad \text{(Eq)}$$

Pretoze dominantna zlozka signalu pri aerosolovom rozptyle pochadza z dopredneho rozptylu, mozeme spodnu hranicu integrovania obmedzit na nulu (teda z=0..pi/2) a potom dostavame:

$$I_A(z) \propto \frac{Q_0}{\cos z} \frac{1}{D} \frac{k_A}{4\pi} \frac{1}{(1-g)} \left\{ E\left(\frac{\pi}{4}+\frac{z}{2}, \frac{4g}{(1+g)^2}\right) - E\left(\frac{z}{2}, \frac{4g}{(1+g)^2}\right) + \right. \\ \left. + \frac{2g}{(1+g)} \left[\frac{\sin z}{\sqrt{1+g^2+2g\cos z}} - \frac{\cos z}{\sqrt{1+g^2-2g\sin z}} \right] \right\} \quad \text{(Eq)}$$

kde E je neuplny elipticky integral, i.e.:

$$E(\varphi, x^2) = \int_0^{\varphi} \sqrt{1-x^2\sin^2\vartheta}\; d\vartheta \quad \text{(Eq)}$$

Da sa ukazat, ze:

$$E\left(\frac{\pi}{4}+\frac{z}{2}, \xi\right) - E\left(\frac{z}{2}, \xi\right) \\ = E(\xi) - E\left(\frac{\pi}{4}, \xi\right) + \frac{\left(\frac{\pi}{2}-z\right)}{2(1+g)} \left[\sqrt{1+g^2} + g - 1 + \left(\frac{\pi}{2}-z\right) \frac{g\sqrt{1+g^2}}{(1-g)^2} \right] \quad \text{(Eq)}$$

pricom rozdiel dvoch eliptickych integralov na pravej strane horeuvedenej rovnice vieme rozpisat ako rozvoj, z ktoreho prve styri cleny su:

$$E(\xi) - E\left(\frac{\pi}{4}, \xi\right) \approx \frac{\pi}{4} - \frac{(2+\pi)}{16}\xi - \frac{(8+3\pi)}{256}\xi^2 - \frac{(44+15\pi)}{3072}\xi^3 - \frac{(1600+525\pi)}{196608}\xi^4 \cdots \quad \text{(Eq)}$$

Parameter

$$\xi = \frac{4g}{(1+g)^2} \quad \text{(Eq)}$$

nadobuda hodnoty od 0 do 1, ale pre vacsinu typickych asymmetry parametrov (g) je blizky jednotke. Napriek tomu si myslim, ze rozvoj az po stvrty rad staci, kedze expanzne koeficienty klesaju celkom obstojne:

$$E(\xi) - E\left(\frac{\pi}{4}, \xi\right) \approx 1.57 - 0.32\xi - 0.07\xi^2 - 0.03\xi^3 - 0.016\xi^4 \cdots \quad \text{(Eq)}$$

Napokon:

$$I_A(z) \propto \frac{Q_0}{\cos z} \frac{1}{D} \frac{k_A}{4\pi} \frac{1}{(1-g)} \left\{ E(\xi) - E\left(\frac{\pi}{4}, \xi\right) \right. \\ + \left(\frac{\pi}{2}-z\right) \frac{1}{2(1+g)} \times\times \left[\sqrt{1+g^2} + g - 1 + \left(\frac{\pi}{2}-z\right) \frac{g\sqrt{1+g^2}}{(1-g)^2} \right] + \\ \left. + \frac{2g}{(1+g)} \left[\frac{\sin z}{\sqrt{1+g^2+2g\cos z}} - \frac{\cos z}{\sqrt{1+g^2-2g\sin z}} \right] \right\} \quad \text{(Eq)}$$

kde $E(\xi) - E\left(\frac{\pi}{4}, \xi\right)$ nahradzujeme rozvojom.

Celkovy signal je: $I(z) = I_R(z) + I_A(z)$.

Ak nahradime objemove koeficienty rozptylu optickymi hrubkami (co sa da, kedze su linearne zavisle), tak potom

$$I_R(z) \propto \frac{Q_0}{\cos z}\frac{3}{D}\frac{k_R}{16\pi}\left(\frac{\pi}{4}+\frac{z}{2}+\frac{\sin 2z}{4}\right) \qquad \text{(Eq.1)}$$

$$\begin{aligned} I_A(z) \propto \frac{Q_0}{\cos z}\frac{1}{D}\frac{\tau_A}{4\pi}\frac{1}{(1-g)}\Bigg\{ & E(\xi) - E\left(\frac{\pi}{4},\xi\right) \\ & + \left(\frac{\pi}{2}-z\right)\frac{1}{2(1+g)}\times\times\left[\sqrt{1+g^2}+g-1+\left(\frac{\pi}{2}-z\right)\frac{g\sqrt{1+g^2}}{(1-g)^2}\right]+ \\ & + \frac{2g}{(1+g)}\left[\frac{\sin z}{\sqrt{1+g^2+2g\cos z}}-\frac{\cos z}{\sqrt{1+g^2-2g\sin z}}\right]\Bigg\} \end{aligned} \qquad \text{(Eq.2)}$$

kde spektralnu zavislost simulujeme v tychto parametroch nasledovne:

$\tau_R = 0.00879\lambda^{-4.09}$ (λ v mikrometroch)

$\tau_A = \tau_{A0}\left(\frac{\lambda}{\lambda_0}\right)^{-\alpha}$ (kde τ_{A0} je aerosolova opticka hrubka na vlnovej dlzke λ_0)

$g = \frac{\cos^2 G(a,\lambda b)}{1+G^2(a,\lambda b)}$ (kde a a b su parametre modifikovanej distribucnej funkcie castic, f(r))

$G(a,\lambda b) = \frac{10\gamma^2+\lambda b}{8\pi}\frac{8}{10+5\ \ (\gamma\pi)^{-2}}$ (kde γ=0.577 je Eulerova konstanta). má tam by "a" v menovateli veda 5

Volbou parametrov a, b, τ_{A0} pri danej λ_0, α a zenitoveho uhla pozorovania z mozeme simulovat jas oblohy nad mestom ako $I(z) = I_R(z) + I_A(z)$. Q_0 je konstanta umernosti (mnozstvo fotonov emitovanych do jednotkoveho priestoroveho uhla) a D je vzdialenost pozorovatela od zdroja.

Mozeme napr. volit: $a = 2$, $b = 20\ [\mu m^{-1}]$, $\tau_{A0} = 0.1 \ldots 0.4$ pri danej $\lambda_0 = 0.5\ [\mu m]$, $\alpha = 1$