Matrix models of fuzzy field theories

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cost Action MP 1405
Quantum Structure of Spacetime

Quantum Spacetime and Physics Models, 18.9.2017, Corfu

Math - Matrix models

- ensemble of matrices, probability measure
- expectation values, correlation functions, partition function
- eigenvalue distribution
- a good tool to analyse (some) properties of fuzzy field theories

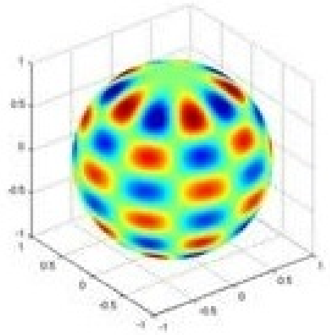
Physics - fuzzy field theory

- (compact) noncommutative space, (real scalar) field theory
- (naïve) commutative limit of NC theory is different from commutative theory - UV/IR mixing
- different spontaneous symmetry breaking patterns

Physics - fuzzy field theory

- Noncommutative spaces introduce a shortest possible distance.
- Fuzzy spaces (= a finite dimensional algebra) have finite number of the "Planck cells" N.
- The hallmark example is the fuzzy sphere S^2_F.

  Hoppe '82; Madore '92; Grosse, Klimcik, Presnajder '90s
- However there are no sharp boundaries between the pieces and everything is blurred, or fuzzy.



Physics - fuzzy field theory

Balachandran, Kürkçüoğlu, Vaidya '05; Szabo '03

- **Commutative** euclidean theory of a real scalar field is given by an action

$$S(\Phi) = \int dx \left[\frac{1}{2} \Phi \Delta \Phi + \frac{1}{2} m^2 \Phi^2 + V(\Phi) \right]$$

  and path integral correlation functions

$$\langle F \rangle = \frac{\int d\Phi \, F(\Phi) e^{-S(\Phi)}}{\int d\Phi \, e^{-S(\Phi)}} \ .$$

- **Noncommutative** euclidean theory of a real scalar field given by an action (for S_F^2)

$$S(M) = \frac{4\pi R^2}{N} \mathrm{Tr} \left[\frac{1}{2} M \frac{1}{R^2} [L_i, [L_i, M]] + \frac{1}{2} m^2 M^2 + V(M) \right]$$

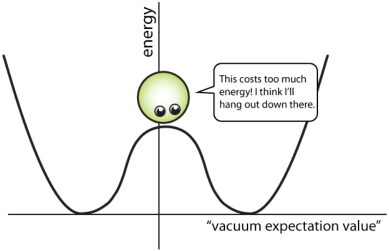
  and path integral correlation functions

$$\langle F \rangle = \frac{\int dM \, F(M) e^{-S(M)}}{\int dM \, e^{-S(M)}} \ .$$

- Eigenvalues of the matrix correspond to values of the field on the "cells" of the space.

Physics - fuzzy field theory

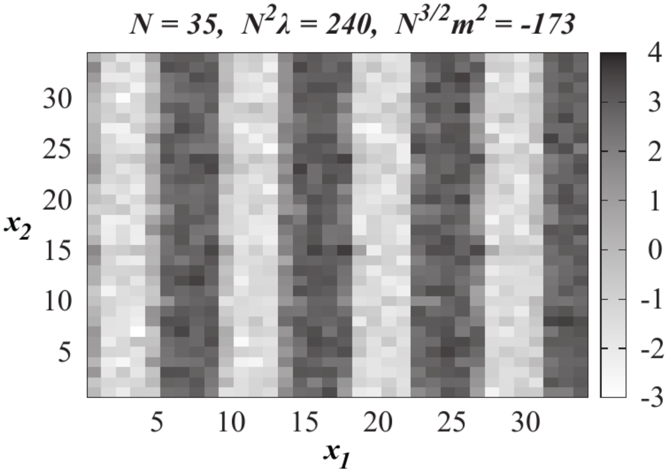
- **The commutative** theory has two phases.
  Glimm, Jaffe '74; Glimm, Jaffe, Spencer '75; Chang '76
  Loinaz, Willey '98; Schaich, Loinaz '09



- **The noncommutative** theory has one more phase.
  Gubser, Sondhi '01; G.-H. Chen and Y.-S. Wu '02
  Martin '04; García Flores, Martin, O'Connor '06, '09; Panero '06, '07; Ydri '14;
  Bietenholz, F. Hofheinz, Mejía-Díaz, Panero '14; Mejía-Díaz, Bietenholz, Panero '14;
  Medina, Bietenholz, D. O'Connor '08; Bietenholz, Hofheinz, Nishimura '04; Lizzi, Spisso '12; Ydri, Ramda, Rouag '16
  Panero CORFU2015

Physics - fuzzy field theory

Mejía-Díaz, Bietenholz, Panero '14



Math - Matrix models

- Ensemble of hermitian $N \times N$ matrices with a probability measure $S(M)$ and expectation values

$$\langle F \rangle = \frac{\int dM\, F(M) e^{-S(M)}}{\int dM\, e^{-S(M)}} \ .$$

- This is the very same expression as for the real scalar field.
- Fuzzy field theory = matrix model with

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2} r\, \mathrm{Tr}\left(M^2\right) + g\, \mathrm{Tr}\left(M^4\right)$$

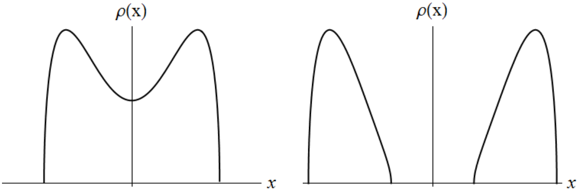
(minus the red Brezin, Itzykson, Parisi, Zuber '78)

Math - Matrix models

- The model **without** kinetic term

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)$$

  is **well** understood.
- The key keywords are diagonalization and large N limit.
- The key results is that for $r < -4\sqrt{g}$ we get two cut eigenvalue density.



Math - Matrix models

- The model **with** kinetic term

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)$$

  is **not well** understood.

  Steinacker '05; JT Acta Physica Slovaca '15

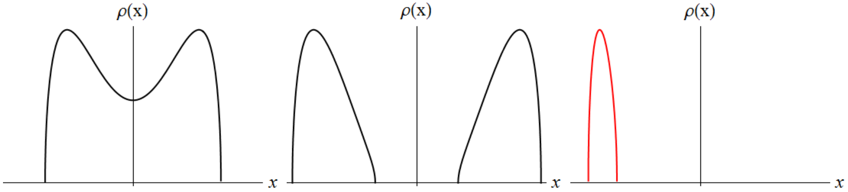
- The key issue being that diagonalization no longer straightforward.
- All previous or current approaches are based on an effective action

$$S(M) \to S_{eff}(x_i) + \frac{1}{2}r\sum_i x_i^2 + g\sum_i x_i^4 - 2\sum_{i<j}\log|x_i - x_j|\ ,$$

  but only approximations to $S_{eff}(x_i)$ are known.

Math - Matrix models

- There are some promising nonperturbative results for S_F^2.
  work in progress
- Most importantly existence of an asymmetric one cut phase, corresponding to the "standard" symmetry broken phase.



- The results are in a(n unexpectedly) good agreement with numerical simulations.
  work in progress by O'Connor, Kovacik

Outlook

To do list.

- Find (a more) complete understanding of the matrix model.
- Investigate matrix models corresponding to spaces beyond the fuzzy sphere.
- Investigate matrix models corresponding to theories without the UV/IR mixing. The kinetic term should completely remove the matrix phase.

Thank you for your attention!

Summary of different approaches to $S_{eff}(x_i)$

$$e^{-S_{eff}(\lambda_i)} = \int dU\, e^{-\frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger [L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

- Perturbative - expanding in powers of the kinetic term, yields multitrace model, kinetic term large in the interesting cases = perturbative approach fails (badly)

  O'Connor, Sämann '07; Sämann '10; Sämann '15; Rea, Sämann '15; Ydri '16
- Nonperturbative

$$S_{eff} = \frac{1}{2}F(c_2 - c_1^2) + \mathcal{R} \quad , \quad F(t) = \log\left(\frac{t}{1-e^{-t}}\right)$$

  Steinacker '05; JT '13; Polychronakos '13; JT '14, JT '15, work in progress
- Two body interaction

$$S_{eff} = \sum_{i,j} a \log|1 - b\, x_i x_j|$$

  work in progress with M. Šubjaková