Matrix models of fuzzy field theories

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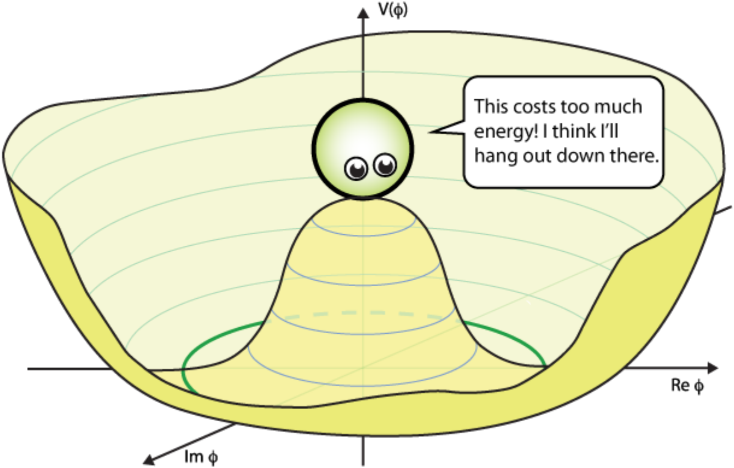
Math - Matrix models

- ensemble of matrices, probability measure
- expectation values, correlation functions, partition function
- eigenvalue distribution
- a good tool to analyse (some) properties of fuzzy field theories

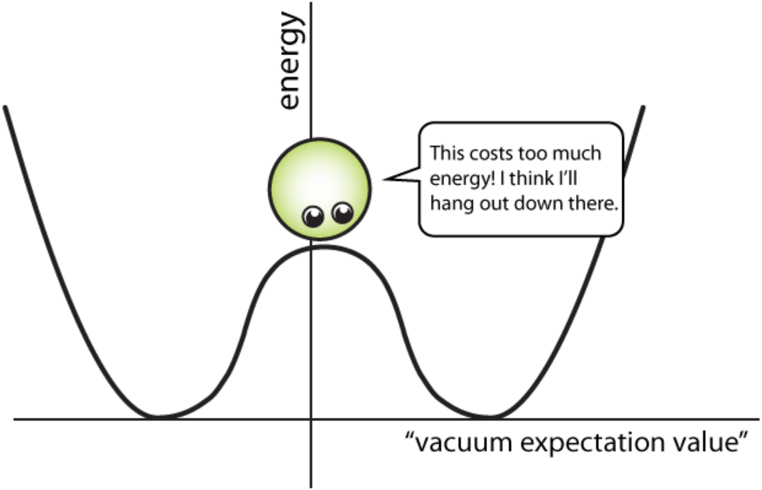
Physics - fuzzy field theory

- (compact) noncommutative space, (real scalar) field theory
- (naïve) commutative limit of NC theory is different from commutative theory - UV/IR mixing
- different spontaneous symmetry breaking patterns

The ABEGHHK'tH mechanism



Real scalar ϕ^4 field on plane



Introduction and outline

quantum theory + general relativity

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?!?!?!

Introduction and outline

quantum theory + general relativity

$\Downarrow$

some nontrivial short distance structure of space

Doplicher, Fredenhagen, Roberststs '95

Introduction and outline

- To measure an event of spatial extent Δx, we need a particle with a similar wavelength. According to de Broglie, this particle has energy

$$E \sim \frac{1}{\Delta x} \ .$$

- As we lower Δx beyond a certain point, the concentration of energy will create a black hole. The result of the measurement will be hidden under the event horizon of this black hole and we can not obtain the information we were after.
- Rather unsurprisingly, this will happen at the Planck scale

$$R_S = \frac{2GM}{c^2} \ , \ E = Mc^2 \ , \ E = \frac{hc}{\lambda} \ \Rightarrow \ L = \sqrt{2} \underbrace{\sqrt{\frac{hG}{c^3}}}_{l_{pl}} \ .$$

Introduction and outline

quantum theory + general relativity

$\Downarrow$

some nontrivial short distance structure of space

$\Uparrow$

space noncommutatitvity

Introduction and outline

In this talk, we will

- describe the construction of the fuzzy sphere S_F^2, as the trademark example of a noncommutative space,
- show how the noncommutativity introduces a short distance structure without any loss of symmetry of the space,
- mention some most interesting properties of scalar field theories defined on such spaces,
- describe this theory in terms of a random matrix model,
- investigate the properties of this model.

Introduction and outline

Take home message.

- Fuzzy (and NC) spaces have a discrete, yet symmetric, short distance structure.
- This comes with some unexpected consequences.
- Matrix models are a great tool to analyze the properties of (scalar field theories on) fuzzy spaces.

Construction(s) of fuzzy spaces

Fuzzy sphere Hoppe '82; Madore '92; Grosse, Klimčík, Prešnajder '90s; Steinacker '13

- Functions on the usual sphere are given by

$$f(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta,\phi) \ ,$$

where Y_{lm} are the spherical harmonics

$$\Delta Y_{lm}(\theta,\phi) = l(l+1) Y_{lm}(\theta,\phi) \ .$$

- They form an (commutative) algebra and all the information about the sphere is encoded in this algebra (Gelfand, Naimark).
- To describe features at a length scale δx we need functions Y_m^l with

$$l \sim \frac{1}{\delta x} \ .$$

Fuzzy sphere

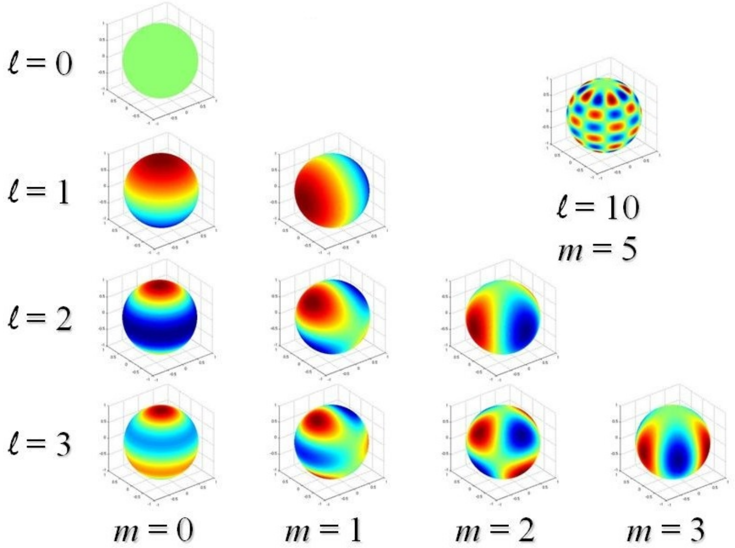


Image taken from http://principles.ou.edu/mag/earth.html

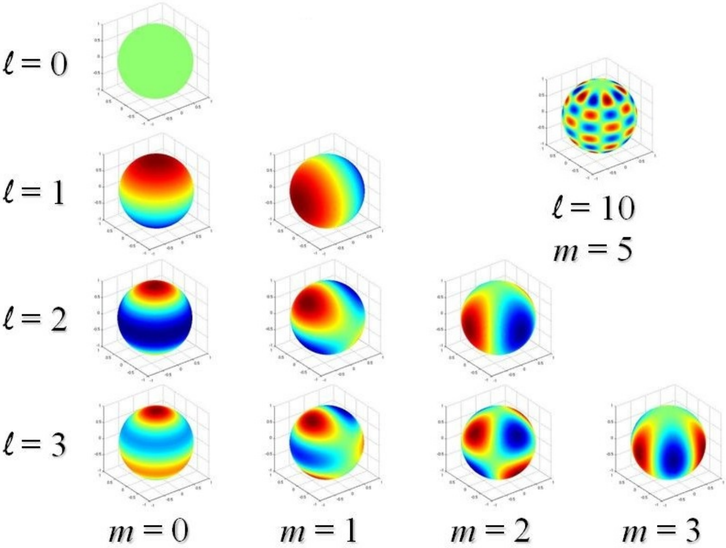
Fuzzy sphere

- If we truncate the possible values of l in the expansion

$$f = \sum_{l=0}^{L} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi) \ ,$$

  we will not be able to see any features of functions at length scales under $\sim 1/L$.
- Points on the sphere (as δ-functions) cease to exist.
- Expressions defined in this way are not closed under multiplication.

Fuzzy sphere



Fuzzy sphere

- Number of independent functions with $l \leq L$ is

$$\sum_{l=0}^{L} \sum_{m=-l}^{l} 1 = (L+1)^2 = N^2 \ .$$

  This is the same as the number of $N \times N$ hermitian matrices

$$N + 2\sum_{n=1}^{N}(n-1) = N^2 \ .$$

  The idea is to map the former on the latter and borrow a closed product from there.

- In order to do so, we consider a $N \times N$ matrix as a product of two N-dimensional representations $\underline{N}$ of the group $SU(2)$. It reduces to

$$\begin{array}{ccccccccc} \underline{N} \otimes \underline{N} & = & \underline{1} & \oplus & \underline{3} & \oplus & \underline{5} & \oplus & \ldots \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & = & \{Y_{0m}\} & \oplus & \{Y_{1m}\} & \oplus & \{Y_{2m}\} & \oplus & \ldots \end{array}$$

Fuzzy sphere

- We thus have a map $\varphi : Y_{lm} \to M$ and we define the product

$$Y_{lm} * Y_{l'm'} := \varphi^{-1} \left(\varphi \left(Y_{lm} \right) \varphi \left(Y_{l'm'} \right) \right) \ .$$

- We have obtained a short distance structure, but the prize we had to pay was a noncommutative product $*$ of functions. The space, for which this is the algebra of functions, is called the fuzzy sphere.
- Opposing to some lattice discretization this space still possess a full rotational symmetry.
- In the limit $N \to \infty$ we recover the original sphere.

Fuzzy sphere - an alternative

- The regualar sphere S^2 is given by the coordinates

$$x_i x_i = R^2 \quad , \quad x_i x_j - x_j x_i = 0 \; ,$$

which generate the following algebra of functions

$$f = \left\{ \sum_{k\in\mathbb{N}^3} \left(a_{k_1 k_2 k_3} \prod_{i=1}^{3} x_i^{k_i} \right) \middle| x_i x_i = R^2 \right\} \; ,$$

which is by definition commutative.

- Information about the sphere is again hidden in this algebra.

Fuzzy sphere - an alternative

- For the fuzzy sphere S_F^2 we define

$$x_i x_i = \rho^2 \quad , \quad x_i x_j - x_j x_i = i\theta \varepsilon_{ijk} x_k \ .$$

- Such x_i's generate a different, noncommutative algebra and S_F^2 is an object, which has this algebra as an algebra of functions.
- The conditions can be realized as an $N = 2j + 1$ dimensional representation of $SU(2)$

$$x_i = \frac{2r}{\sqrt{N^2 - 1}} L_i \quad , \quad \theta = \frac{2r}{\sqrt{N^2 - 1}} \quad , \quad \rho^2 = \frac{4r^2}{N^2 - 1} j(j+1) = r^2 \ .$$

- The group $SU(2)$ still acts on x_i's and this space enjoys a full rotational symmetry.
- And again, in the limit $N \to \infty$ we recover the original sphere.

Fuzzy sphere - an alternative

- x_i's are $N \times N$ matrices, functions on S_F^2 are combinations of all their possible products and thus hermitian matrices M.
- Such $N \times N$ matrix can be decomposed into

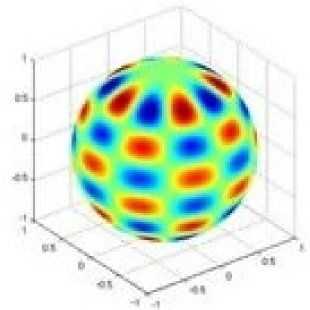
$$M = \sum_{l=0}^{N-1} \sum_{m=-l}^{m=l} c_{lm} T_{lm} \ .$$

where matrices T_m^l are called polarization tensors and

$$\begin{aligned} T_m^l &= \varphi(Y_m^l) \ , \\ \mathrm{Tr}\,(T_{lm} T_{l'm'}) &= \delta_{ll'} \delta_{mm'} \ , \\ [L_i, [L_i, T_{lm}]] &= l(l+1) T_{lm} \ . \end{aligned}$$

Fuzzy sphere - conclusion

- Either way, we have divided the sphere into N cells. Function on the fuzzy sphere is given by a matrix M and the eigenvalues of M represent the values of the function on these cells.



- However there are no sharp boundaries between the pieces and everything is blurred, or fuzzy.

Fuzzy spaces in physics

- Regularization of infinities in the standard QFT.
  Heisenberg ∼'30; Snyder '47, Yang '47
- Regularization of field theories for numerical simulations.
  Panero '16
- An effective description of the open string dynamics in a magnetic background in the low energy limit.
  Seiberg Witten '99; Douglas, Nekrasov '01
- Solutions of various matrix formulations of the string theory.
  Steinacker '13
- Geometric unification of the particle physics and theory of gravity.
  van Suijlekom '15
- An effective description of various systems in a certain limit (eg. QHE).
  Karabali, Nair '06

Fuzzy scalar field theory

Scalar field theory

- Commutative euclidean theory of a real scalar field is given by an action

$$S(\Phi) = \int dx \left[\frac{1}{2}\Phi\Delta\Phi + \frac{1}{2}m^2\Phi^2 + V(\Phi) \right]$$

and path integral correlation functions

$$\langle F \rangle = \frac{\int d\Phi\, F(\Phi) e^{-S(\Phi)}}{\int d\Phi\, e^{-S(\Phi)}} \ .$$

- We construct the noncommutative theory as an analogue with
  - field $\rightarrow$ matrix,
  - functional integral $\rightarrow$ matrix integral,
  - spacetime integral $\rightarrow$ trace,
  - derivative $\rightarrow$ L_i commutator.

Scalar field theory

- **Commutative**

$$S(\Phi) = \int dx \left[\frac{1}{2} \Phi \Delta \Phi + \frac{1}{2} m^2 \Phi^2 + V(\Phi) \right]$$

$$\langle F \rangle = \frac{\int d\Phi \, F(\Phi) e^{-S(\Phi)}}{\int d\Phi \, e^{-S(\Phi)}} \; .$$

- **Noncommutative** (for S_F^2)

$$S(M) = \frac{4\pi R^2}{N} \mathrm{Tr} \left[\frac{1}{2} M \frac{1}{R^2} [L_i, [L_i, M]] + \frac{1}{2} m^2 M^2 + V(M) \right]$$

$$\langle F \rangle = \frac{\int dM \, F(M) e^{-S(M)}}{\int dM \, e^{-S(M)}} \; .$$

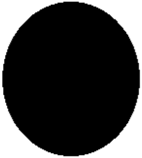
Balachandran, Kürkçüoğlu, Vaidya ’05; Szabo ’03

UV/IR mixing

UV/IR mixing

- The key property of the noncommutative field theories is the UV/IR mixing phenomenon, which arises as a result of the nonlocality of the theory.

  Minwalla, Van Raamsdonk, Seiberg '00; Chu, Madore, Steinacker '01
- Quanta can not be compressed into an arbitrarily small volume. If we try to squeze a packet in one direction, it will spread out in a different one. Processes with large momentum contribute to processes at small momentum.



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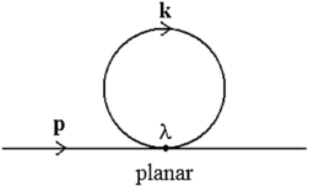
UV/IR mixing

- In terms of diagrams different properties of planar and non-planar ones.



- Bad behaviour of higher loop diagrams.

UV/IR mixing

- There is no clear separation of scales and the theory is no longer renormalizable.
- This effect survives the commutative limit.
- The commutative limit of a noncommutative theory is very different from the commutative theory we started with.
- The space (geometry) forgets where it came from but the field theory (physics) remembers its fuzzy origin.

Spontaneous symmetry breaking

Symmetry breaking in NC field theories

- The commutative field theory has two phases in the phase diagram, disorder and uniform order phases.
  Glimm, Jaffe '74; Glimm, Jaffe, Spencer '75; Chang '76
  Loinaz, Willey '98; Schaich, Loinaz '09
- In disorder phase the field oscillates around the value $\phi = 0$.
- In uniform order phase the field oscillates around a nonzero value which is a minimum of the potential.

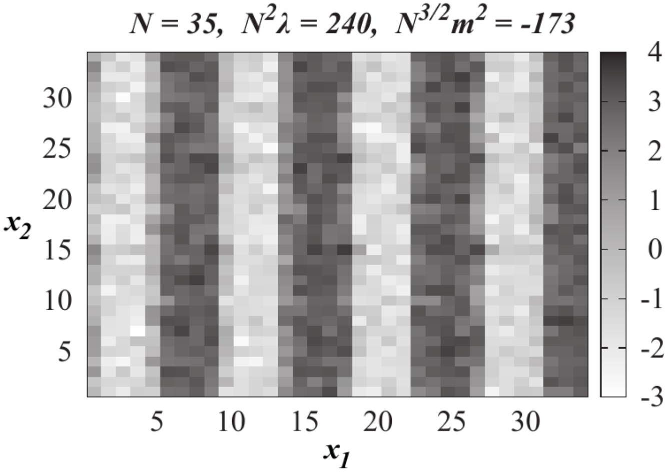
Symmetry breaking in NC field theories

Symmetry breaking in NC field theories

- The phase diagram of noncommutative field theories has one more phase. It is a non-uniform order phase, or a striped phase.
  Gubser, Sondhi '01; G.-H. Chen and Y.-S. Wu '02
- In this phase, the field does not oscillate around one given value in the whole space. Translational symmetry is broken.
- This has been established in numerous numerical works for variety different spaces.
  Martin '04; García Flores, Martin, O'Connor '06, '09; Panero '06, '07; Ydri '14; Bietenholz, F. Hofheinz, Mejía-Díaz, Panero '14; Mejía-Díaz, Bietenholz, Panero '14; Medina, Bietenholz, D. O'Connor '08; Bietenholz, Hofheinz, Nishimura '04; Lizzi, Spisso '12; Ydri, Ramda, Rouag '16
  Panero '15

Symmetry breaking in NC field theories

Mejía-Díaz, Bietenholz, Panero '14 for $\mathbb{R}^2_\theta$

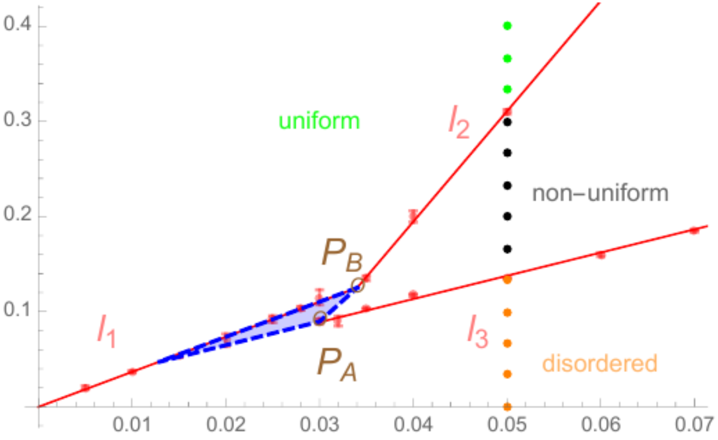


Symmetry breaking in NC field theories

- This phase is a result of the nonlocality of the theory.
  Mermin-Wagner Theorem : no spontaneous breaking of a continuous symmetry in local 2-dimensional theories.
- This phase survives the commutative limit of the noncommutative theory! Result of the UV/IR mixing.
- The commutative limit of such noncommutative theory is (even more) different than the commutative theory we started with.

Symmetry breaking in NC field theories

O'Connor, Kováčik '18 for S_F^2



Matrix model description of fuzzy field theories

Matrix models

- Ensemble of hermitian $N \times N$ matrices with a probability measure $S(M)$ and expectation values

$$\langle F \rangle = \frac{\int dM\, F(M) e^{-S(M)}}{\int dM\, e^{-S(M)}} \ .$$

- This is the very same expression as for the real scalar field.
- Fuzzy field theory = matrix model with

$$S(M) = \frac{1}{2} \mathrm{Tr} \left(M [L_i, [L_i, M]] \right) + \frac{1}{2} r \,\mathrm{Tr} \left(M^2 \right) + g \,\mathrm{Tr} \left(M^4 \right)$$

(minus the red Brezin, Itzykson, Parisi, Zuber '78)

Matrix models

- Matrix model with

$$S(M) = \frac{1}{2} r \mathrm{Tr}\left(M^2\right) + g \mathrm{Tr}\left(M^4\right)$$

- We diagonalize $M = U \Lambda U^\dagger$ for some $U \in SU(N)$ and $\Lambda = diag(\lambda_1, \ldots, \lambda_N)$, the integration measure becomes

$$dM = dU \left(\prod_{i=1}^{N} d\lambda_i \right) \times \prod_{i<j} (\lambda_i - \lambda_j)^2$$

and we are to compute integrals like

$$\langle F \rangle \sim \int \left(\prod_{i=1}^{N} d\lambda_i \right) F(\lambda_i)\, e^{-N^2 \left[\frac{1}{2} r \frac{1}{N} \sum \lambda_i^2 + g \frac{1}{N} \sum \lambda_i^4 - \frac{2}{N^2} \sum_{i<j} \log |\lambda_i - \lambda_j| \right]}$$

$$\times \int dU$$

Matrix models

- In the $N \to \infty$ limit the probability measure localizes on configurations minimizing the expression

$$\frac{1}{2} r \frac{1}{N} \sum \lambda_i^2 + g \frac{1}{N} \sum \lambda_i^4 - \frac{2}{N^2} \sum_{i<j} \log |\lambda_i - \lambda_j|$$

- Equivalent to a 1D gas of N repelling particles.
- Several technical steps, introduction of a continuous eigenvalue distribution $\rho(x)$ and resolvent

$$\omega(z) = \int dy \frac{\rho(y)}{z-y} \ ,$$

solution of a Riemann-Hilbert problem

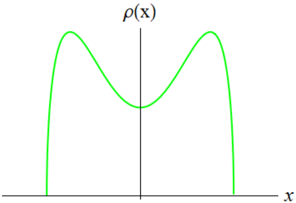
$$\omega(x + i0^+) + \omega(x - i0^+) = rx + V'(x) \ , \ x \in \text{supp}\, \rho$$

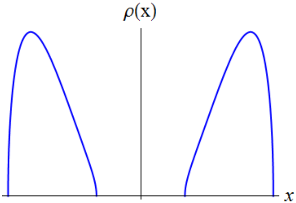
Matrix models of fuzzy field theories

- The model **without** the kinetic term

$$S(M) = \frac{1}{2}\text{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\,\text{Tr}\left(M^2\right) + g\,\text{Tr}\left(M^4\right)$$

  is **well** understood.
- The key results is that for $r < -4\sqrt{g}$ we get two cut eigenvalue density.





Matrix models of fuzzy field theories

- The model **with** the kinetic term

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2} r\,\mathrm{Tr}\left(M^2\right) + g\,\mathrm{Tr}\left(M^4\right)$$

  is **not well** understood.

  Steinacker '05; JT Acta Physica Slovaca '15

- The key issue being that diagonalization no longer straightforward.

Matrix models of fuzzy field theories

- Fuzzy field theory = matrix model with

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\mathrm{Tr}\left(M^2\right) + g\mathrm{Tr}\left(M^4\right)$$

- We are to compute integrals like

$$\langle F\rangle \sim \int\left(\prod_{i=1}^N d\lambda_i\right)F(\lambda_i)\, e^{-N^2\left[\frac{1}{2}r\frac{1}{N}\sum\lambda_i^2+g\frac{1}{N}\sum\lambda_i^4-\frac{2}{N^2}\sum_{i<j}\log|\lambda_i-\lambda_j|\right]}$$

$$\times\int dU\, e^{-N^2\frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger[L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

Matrix models of fuzzy field theories

- Fuzzy field theory = matrix model with

$$S(M) = \frac{1}{2}\mathrm{Tr}\left(M[L_i,[L_i,M]]\right) + \frac{1}{2}r\mathrm{Tr}\left(M^2\right) + g\mathrm{Tr}\left(M^4\right)$$

- We are to compute integrals like

$$\langle F\rangle \sim \int \left(\prod_{i=1}^{N} d\lambda_i\right) F(\lambda_i)\, e^{-N^2\left[S_{eff}(\lambda_i)+\frac{1}{2}r\frac{1}{N}\sum\lambda_i^2+g\frac{1}{N}\sum\lambda_i^4-\frac{2}{N^2}\sum_{i<j}\log|\lambda_i-\lambda_j|\right]}$$

$$e^{-N^2 S_{eff}(\lambda_i)} = \int dU\, e^{-N^2\frac{1}{2}\mathrm{Tr}\left(U\Lambda U^\dagger[L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

Steinacker '05

- How to compute S_{eff}?

Matrix models of fuzzy field theories

- Perturbative calculation of the integral show that the S_{eff} contains products of traces of M. O'Connor, Sämann '07; Sämann '10

$$e^{-N^2 S_{eff}(\lambda_i)} = \int dU \, e^{-N^2 \varepsilon \frac{1}{2} \mathrm{Tr}\left(U\Lambda U^\dagger [L_i,[L_i,U\Lambda U^\dagger]]\right)}$$

- The most recent result is Sämann '15

$$S_{eff}(M) = \frac{1}{2}\left[\varepsilon \frac{1}{2}\left(c_2 - c_1^2\right) - \varepsilon^2 \frac{1}{24}\left(c_2 - c_1^2\right)^2 + \varepsilon^4 \frac{1}{2880}\left(c_2 - c_1^2\right)^4\right] - \\ - \varepsilon^4 \frac{1}{3456}\left[\left(c_4 - 4c_3c_1 + 6c_2c_1^2 - 3c_1^4\right) - 2\left(c_2 - c_1^2\right)^2\right]^2 - \\ - \varepsilon^3 \frac{1}{432}\left[c_3 - 3c_1c_2 + 2c_1^3\right]^2$$

where

$$c_n = \frac{1}{N}\mathrm{Tr}\left(M^n\right)$$

- Standard treatment of such multitrace matrix model yields a very unpleasant behaviour. Self interaction is way too strong in the important region.

Hermitian matrix model of fuzzy field theories

- For the free theory $g = 0$ the kinetic term just rescales the eigenvalues. Steinacker '05
- There is a unique parameter independent effective action that reconstructs this rescaling. Polychronakos '13

$$S_{eff} = \frac{1}{2} F(c_2) + \mathcal{R} = \frac{1}{2} \log\left(\frac{c_2}{1 - e^{-c_2}}\right) + \mathcal{R}$$

- Recall the perturbative action

$$\begin{aligned} S_{eff}(M) = & \frac{1}{2}\left[\varepsilon \frac{1}{2}\left(c_2 - c_1^2\right) - \varepsilon^2 \frac{1}{24}\left(c_2 - c_1^2\right)^2 + \varepsilon^4 \frac{1}{2880}\left(c_2 - c_1^2\right)^4\right] - \\ & - \varepsilon^4 \frac{1}{3456}\left[\left(c_4 - 4c_3c_1 + 6c_2c_1^2 - 3c_1^4\right) - 2\left(c_2 - c_1^2\right)^2\right]^2 - \\ & - \varepsilon^3 \frac{1}{432}\left[c_3 - 3c_1c_2 + 2c_1^3\right]^2 \end{aligned}$$

The first line is the first terms of the small c_2 expansion with $c_2 \to c_2 - c_1^2$.

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- For the free theory $g = 0$ the kinetic term just rescales the eigenvalues. Steinacker '05
- There is a unique parameter independent effective action that reconstructs this rescaling. Polychronakos '13

$$S_{eff} = \frac{1}{2}F(c_2) + \mathcal{R} = \frac{1}{2}\log\left(\frac{c_2}{1 - e^{-c_2}}\right) + \mathcal{R}$$

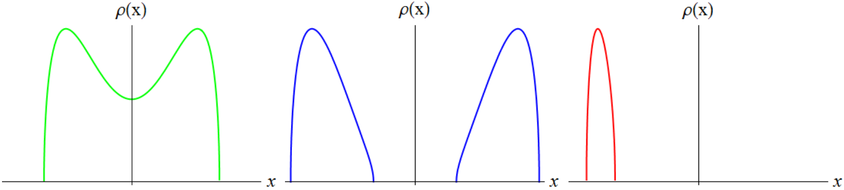
- Introducing the asymmetry $c_2 \to c_2 - c_1^2$ we obtain a matrix model

$$S(M) = \frac{1}{2}F(c_2 - c_1^2) + \frac{1}{2}r, \mathrm{Tr}\left(M^2\right) + g, \mathrm{Tr}\left(M^4\right) \quad , \quad F(t) = \log\left(\frac{t}{1 - e^{-t}}\right)$$

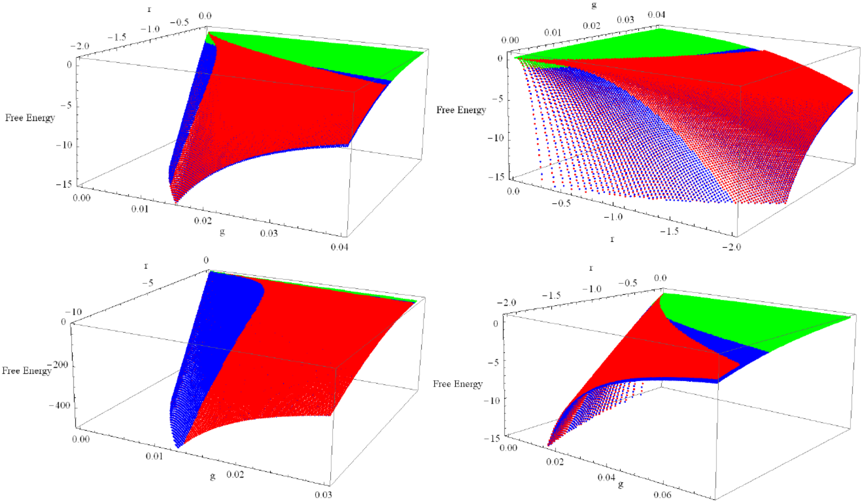
Polychronakos '13; JT '15, JT '17

Matrix models of fuzzy field theories

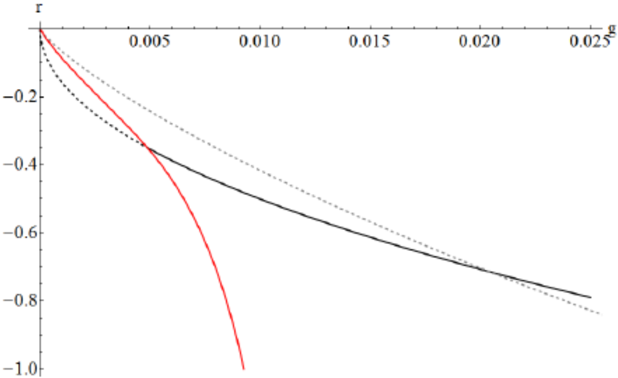
- Such F introduces a (not too strong) interaction among the eigenvalues. For some values of r, g an asymmetric configuration can become stable.
- It corresponds to the ”standard” symmetry broken phase.



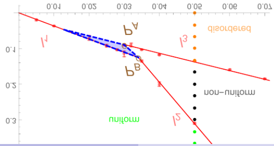
Matrix models of fuzzy field theories



Hermitian matrix model



JT '17



- A very good qualitative agreement. A very good quantitative agreement in the critical coupling.
- Different value for the critical mass parameter and different behaviour of the asymmetric transition line for large $-r$.
- We need to include $\mathcal{R}$ in a nonperturbative way.
  work in progress with M. Šubjaková

- Recall the perturbative action

$$S_{eff}(M) = \frac{1}{2}\left[\varepsilon\frac{1}{2}\left(c_2 - c_1^2\right) - \varepsilon^2\frac{1}{24}\left(c_2 - c_1^2\right)^2 + \varepsilon^4\frac{1}{2880}\left(c_2 - c_1^2\right)^4\right] -$$
$$- \varepsilon^4\frac{1}{3456}\left[\left(c_4 - 4c_3c_1 + 6c_2c_1^2 - 3c_1^4\right) - 2\left(c_2 - c_1^2\right)^2\right]^2 -$$
$$- \varepsilon^3\frac{1}{432}\left[c_3 - 3c_1c_2 + 2c_1^3\right]^2$$

- Find a function which gives a correct perturbative expansion and behaves well close to the triple point. Eg.

$$\log\left(1 + At^2\right)\ ,\ \frac{1}{1 + At^2}\ .$$

- Recall the perturbative action

$$S_{eff}(M) = \frac{1}{2}\left[\varepsilon\frac{1}{2}\left(c_2 - c_1^2\right) - \varepsilon^2\frac{1}{24}\left(c_2 - c_1^2\right)^2 + \varepsilon^4\frac{1}{2880}\left(c_2 - c_1^2\right)^4\right] -$$
$$- \varepsilon^4\frac{1}{3456}\left[\left(c_4 - 4c_3c_1 + 6c_2c_1^2\right) - 3c_1^4 - 2\left(c_2 - c_1^2\right)^2\right]^2 -$$
$$- \varepsilon^3\frac{1}{432}\left[c_3 - 3c_1c_2 + 2c_1^3\right]^2$$
$$= \frac{1}{2}\frac{1}{2}c_2 - \frac{1}{4}c_1^2 - \frac{1}{24}c_2^2 - \frac{1}{432}c_3^2 - \frac{1}{3456}c_4^2 + \ldots$$

- This part can be interpreted as an additional two-particle interaction, modifying the Riemann-Hilbert problem.

- Recall the perturbative action

$$S_{eff}(M) = \frac{1}{2}\frac{1}{2}c_2 - \frac{1}{4}c_1^2 - \frac{1}{24}c_2^2 - \frac{1}{432}c_3^2 - \frac{1}{3456}c_4^2 + \dots$$

- Function of the form

$$S_{eff} = \sum_{i,j} a \log|1 - b\,\lambda_i\lambda_j|$$

with $a = 3/2, b = 1/6$ correctly reproduces all four known coefficients.

Conclusions

- Fuzzy spaces have a discreet, yet symmetric, short distance structure.
- This comes with some unexpected consequences.
- Matrix models are a great tool to analyze the properties of (scalar field theories on) fuzzy spaces.

Outlook

To do list.

- Find (a more) complete understanding of the matrix model.
- Investigate matrix models corresponding to spaces beyond the fuzzy sphere.
- Investigate matrix models corresponding to theories without the UV/IR mixing.

Thank you for your attention!

If time permits I

Find (a more) complete understanding of the matrix model.

- It can be shown

$$e^{-N^2 S_{eff}(\lambda_i)} = \int dU\, e^{-N^2 \frac{1}{2} \mathrm{Tr}\left(U\Lambda U^\dagger [L_i,[L_i,U\Lambda U^\dagger]]\right)}$$
$$= \int dU\, e^{-N^2 \frac{1}{2} \mathrm{Tr}\left(U\Lambda U^\dagger [E,[E,U\Lambda U^\dagger]]\right)}$$

  for a single matrix E.
  Steinacker '16

- This is a significantly simpler integral to compute and model to consider.

If time permits II

Investigate matrix models corresponding to theories without the UV/IR mixing.

- For a noncommutative theory with no UV/IR mixing, the extra phase should not be present in the commutative limit of the phase diagram.
  - B.P. Dolan, D. O'Connor and P. Prešnajder [arXiv:0109084],
  - H. Grosse and R. Wulkenhaar [arXiv:0401128],
  - R. Gurau, J. Magnen, V. Rivasseau and A. Tanasa [arXiv:0802.0791].
- Understanding the phase diagram of such theories, especially mechanism of the departure the striped phase could teach us a lot technically and conceptually.