A less commutative view of the Standard Model

Juraj Tekel

Department of Theoretical Physics

FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
Comenius University
Bratislava

NA62 Collaboration Meeting, Bratislava, Slovakia, 31.8.2022

Take home message

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.
- A certain way to introduce this structure can be also used to reformulate *the Standard model of particle physics* from "first principles".

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.
- A certain way to introduce this structure can be also used to reformulate *the Standard model of particle physics* from "first principles".
- With some new insights and restrictions along the way.

References

- Ali H. Chamseddine, Alain Connes, Matilde Marcolli, *Gravity and the standard model with neutrino mixing*, Adv. Theor. Math. Phys. 11 (2007) 6, 991-1089.
- Reviews:
    - A. Devastato, M. Kurkov, F. Lizzi, 1906.09583
    - F. Lizzi, 1805.00411
    - W. Van Suijlekom, *Noncommutative Geometry and Particle Physics*, Springer 2015
    - A. Connes, Noncommutative Geometry, Wiley 1994

Standard model and general relativity

Standard model

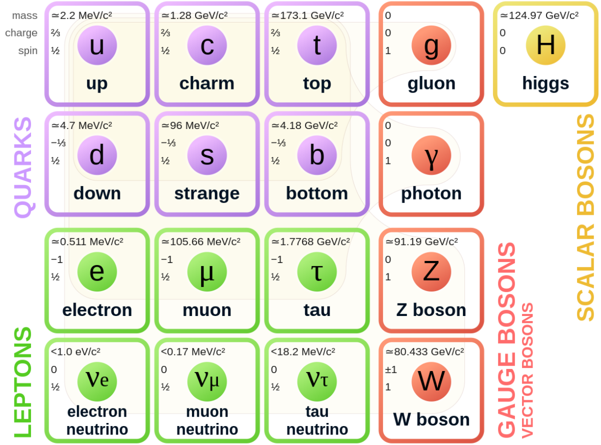


Image from https://commons.wikimedia.org/

Standard model

$$SU(3) \times SU(2) \times U(1)$$

General relativity

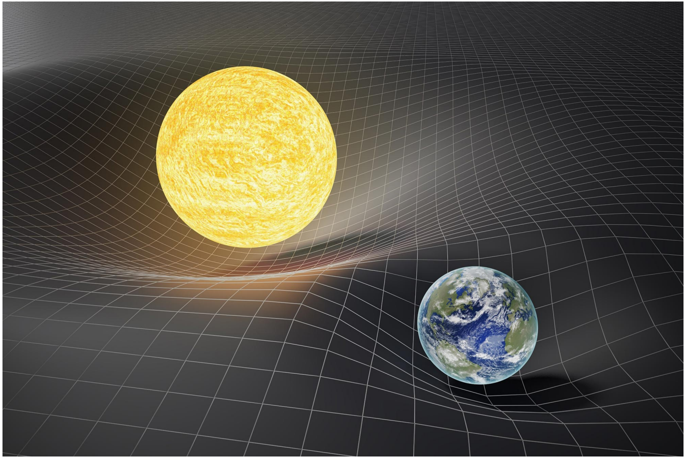


Image from https://scitechdaily.com/

General relativity

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

SM and GR

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} \hat{T}_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} \;??\; \frac{8\pi G}{c^4} \hat{T}_{\mu\nu}$$

Quantum gravity interlude

Quantum structure of spacetime

- We need a quantum theory of gravity.
- Quantization of general relativity leads to a nonrenormalizable theory.
- We have reasons to believe that future theory of quantum gravity will have a different notion of spacetime.
  No distinction between points under certain length scales. [Hossenfelder 1203.6191]
- Reasons:
  - gravitational Heisenberg microscope,
  - emergent spacetime,
  - instability of quantum gravitational vacuum. [Doplicher, Fredenhagen, Roberts '95]

Quantum structure of spacetime

- Very energetic and localized quantum fluctuations can lead to black holes.
- A discrete structure solves this problem.
- Similar to the stabilization of the hydrogen atom in quantum mechanics.

Quantum structure of spacetime

$$\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$$

Quantum structure of spacetime

$$\Delta x \cdot \Delta y \geq \theta$$

- Natural scale for this is $\sqrt{\theta} \approx l_{\mathrm{Pl}} \approx 10^{-35}$.
- A fundamental volume, not length directly.
- Discrete, but preserves at least some of the continuous symmetries.

Noncommutative geometry

NC GEOMETRY

- There is one-to-one correspondence between topological spaces M and commutative $\mathcal{C}^*$ algebras

$$\text{manifold} \;\Leftrightarrow\; \text{functions on the manifold}\ .$$

- Idea of NC geometry is to generalize this notion to noncommutative algebras.
- We need algebra $\mathcal{A}$, hilbert space $\mathcal{H}$ as a representation, an operator D which encodes the geometry of the space

$$D \;\leftrightarrow \sqrt{\Delta}$$

$$D \sim \gamma^\mu \partial_\mu$$

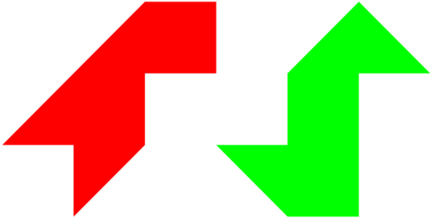
$$\text{geometry} \;\Leftrightarrow\; \text{eigenvalues of } D\ .$$

NC geometry

- Can you hear the shape of a drum?
- Does the spectrum of the laplacian on a manifold determine the geometry?

NC GEOMETRY

- Can you hear the shape of a drum?
- Does the spectrum of the laplacian on a manifold determine the geometry?
- No! [Gordon, Webb, Wolpert, '92]



NC Standard model

NC SM

- SM is described by a particularly simple NC geometry.
- Algebra
$$\mathcal{A} = C(M) \otimes \Big(\underbrace{\mathrm{Mat}_{3\times 3}(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}}_{\mathcal{A}_F} \Big) \ .$$
(The gauge group is given by unitary elements of the algebra.)
- Hilbert space $\mathcal{H}$ is the usual ZOO of SM particles: one generation
  - two kinds of quarks of three colors,
  - two leptons,
  - two chiralities,
  - antiparticles,

  altogether $\mathcal{H} = \mathbb{C}^{96}$.
- The particle content of SM has become part of the the (NC) manifold.

NC SM

- SM is described by a particularly simple NC geometry.
- Algebra

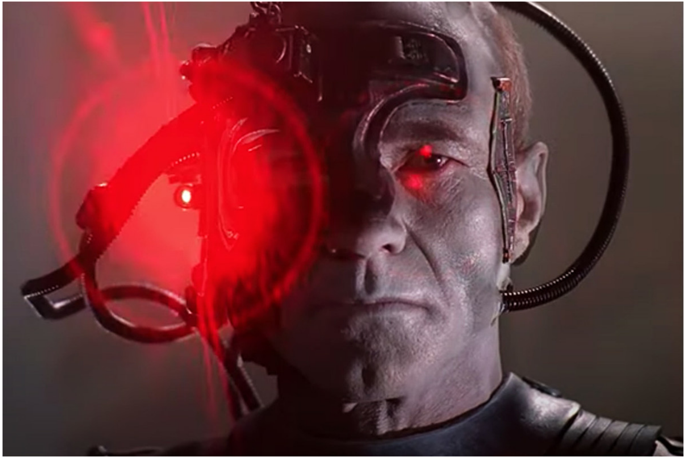
$$\mathcal{A} = C(M) \otimes \Big(\underbrace{\mathrm{Mat}_{3\times 3}(\mathbb{C}) \oplus \mathbb{H} \oplus \mathbb{C}}_{\mathcal{A}_F} \Big) \ .$$

(The gauge group is given by unitary elements of the algebra.)

- Hilbert space $\mathcal{H}$ is the usual ZOO of SM particles: one generation
  - two kinds of quarks of three colors,
  - two leptons,
  - two chiralities,
  - antiparticles,

  altogether $\mathcal{H} = \mathbb{C}^{96}$.
- **The particle content of SM has become part of the the (NC) manifold.**

NC SM



NC SM

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

NC SM

- The Dirac operator is

$$D = \gamma^\mu(\partial_\mu + \omega_\mu) \otimes 1 + \gamma^5 \otimes D_F$$

where D_F is a 96×96 matrix – internal part.

$$D_F = \left[\begin{array}{cccccc|cccccc}
\cdot & \cdot & \Upsilon_\nu & \cdot & \cdot & \cdot & \Upsilon_R^\dagger & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \Upsilon_e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\Upsilon_\nu^\dagger & \Upsilon_e^\dagger & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_u & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \Upsilon_u^\dagger & \Upsilon_d^\dagger & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\hline
\Upsilon_R & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_\nu^* & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_e^* & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_\nu^t & \Upsilon_e^t & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_u^* \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_d^* \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \Upsilon_u^t & \Upsilon_d^t & \cdot
\end{array}\right]$$

NC SM

- Fermions are part of $\mathcal{H}$.
- Fermionic action is given by scalar product

$$\langle \Psi | D\Psi \rangle \ .$$

- This leads to mass terms.

NC SM

- Where are the gauge fields?
- They are given as fluctuations of the Dirac operator!

$$D \to D + A \ .$$

- Possible fluctuations are determined by $\mathcal{A}$
  - $\mathrm{Mat}_{3\times 3}(\mathcal{C})$ gives gluons,
  - $\mathcal{H}$ gives W
  - and $\mathcal{C}$ gives B.
- Fluctuations in the internal space yield the Higgs boson.

Standard model

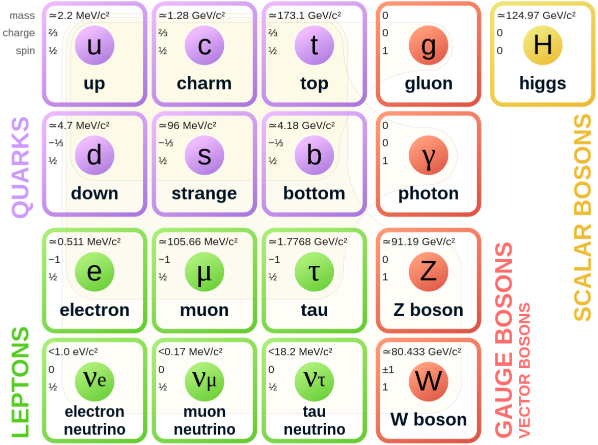


Image from https://commons.wikimedia.org/

NC SM

- All the bosons are now understood on the same footing and coming from the various fluctuations of the Dirac operator.
- Gauge fields are fluctuations of the internal space geometry in a similar way as gravitational fields are fluctuations of the physical space geometry.

NC SM

- All the bosons are now understood on the same footing and coming from the various fluctuations of the Dirac operator.
- **Gauge fields are fluctuations of the internal space geometry in a similar way as gravitational fields are fluctuations of the physical space geometry.**

- Action for bosons is given by

$$\mathrm{Tr}\ \chi\left(\frac{D_A^2}{\Lambda^2}\right) ,$$

where Λ is energy cutoff and χ is a cutoff function.
- "Number of eigenvalues smaller than Λ".
- There are techniques to deal with this expression, e.g. heat kernel.

NC SM

- The mass of the Higgs boson is not a parameter of D_F, only the Yukawa couplings are.
- The vacuum expectation values and the quartic coupling in the Higgs potential are predictions of the model!

NC SM

- The mass of the Higgs boson is not a parameter of D_F, only the Yukawa couplings are.
- The vacuum expectation values and the quartic coupling in the Higgs potential are predictions of the model!
- The predicted mass of the Higgs $\sim 170\ GeV$.

NC SM

- The mass of the Higgs boson is not a parameter of D_F, only the Yukawa couplings are.
- The vacuum expectation values and the quartic coupling in the Higgs potential are predictions of the model!
- **The predicted mass of the Higgs** $\sim 170\ GeV$.

NC SM

- The mass of the Higgs boson is not a parameter of D_F, only the Yukawa couplings are.
- The vacuum expectation values and the quartic coupling in the Higgs potential are predictions of the model!
- **The predicted mass of the Higgs** $\sim 170\ GeV$.

$$170 \neq 125$$

Beyond Standard model

- Considering a more general $\mathcal{A}_F$ in $\mathcal{A}$. Possibilities not that vast.
- We only have matrices over $\mathbb{R}, \mathbb{C}, \mathbb{H}$. The condition on existence of particle-antiparticle pairs and two chiralities yields
$$\mathcal{A}_F = \mathrm{Mat}_{n\times n}(\mathbb{H}) \otimes \mathrm{Mat}_{2n\times 2n}(\mathbb{C}) \ .$$
- The full $n = 2$ case leads to Pati-Salam model, the usual $SU(3)$ is enlarged to $SU(4)$ and hypercharge $U(1)$ to $SU(2)$. Can bring down the Higgs mass to value consistent with experiment.
- There are attempts to make the $n = 4$ case work.
- GUT's are much more strict in this approach, since there are fewer representations of algebras than groups.

Take home message

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.
- A certain way to introduce this structure can be also used to reformulate *the Standard model of particle physics* from "first principles".
- With some new insights and restrictions along the way.

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.
- A certain way to introduce this structure can be also used to reformulate *the Standard model of particle physics* from "first principles".
- With some new insights and restrictions along the way.
  - The particle content of SM has become part of the the NC manifold.
  - Gauge fields are fluctuations of the internal space geometry in a similar way as gravitational fields are fluctuations of the physical space geometry.
  - The predicted mass of the Higgs $\sim 170\, GeV$.

Take home message

- Quantum mechanics and general relativity together lead to a quantum structure of spacetime.
- A certain way to introduce this structure can be also used to reformulate *the Standard model of particle physics* from "first principles".
- With some new insights and restrictions along the way.
    - The particle content of SM has become part of the the NC manifold.
    - Gauge fields are fluctuations of the internal space geometry in a similar way as gravitational fields are fluctuations of the physical space geometry.
    - The predicted mass of the Higgs $\sim 170\, GeV$.

Thank you for your attention!