String modes and fuzzy field theories in string modes formalism

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Faculty of Physics, University of Vienna, 22.3.2022

work with H. Steinacker

arXiv: 2203.02376 [hep-th]

4th Vienna Central European Seminar
on Particle Physics and Quantum Field Theory

"COMMUTATIVE AND NONCOMMUTATIVE QUANTUM FIELDS"

Nov. 30 - Dec. 02, 2007

Faculty of Physics
University of Vienna

Recent developments in relativistic quantum field theory are discussed within the framework of local quantum physics in its algebraic formulation as well as by deforming space time. Aspects of renormalization are dealt with.

Invited speakers: Dorothea Bahns (Hamburg), Detlev Buchholz (Goettingen), Masud Chaichian (Helsinki), Sergio Doplicher (Rome), Stefan Hollands (Goettingen), Gandalf Lechner (Vienna), Vincent Rivasseau (Paris), Philippe Roche (Montpellier), Peter Schupp (Bremen), Richard Szabo (Edinburgh)

Advisory board: Klaus Fredenhagen (Hamburg), Harald Grosse (Vienna), Walter Majerotto (Vienna), Peter Presnajder (Bratislava), Jakob Yngvason (Vienna)

Organizer: H. Hüffel (Vienna)

We encourage junior scientists to apply for participation in this meeting as we received special grants for them.

Further details of the meeting are provided at http://www.univie.ac.at/vienna.seminar/2007/index.html

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Take home message

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- Fuzzy spaces are (among other things) toy models of spaces with quantum structure.
- There is an interesting (new) way to describe functions and operators on fuzzy spaces.
- The description works in position space and uses the coherent states also as a basis for the functions and operators.

Take home message

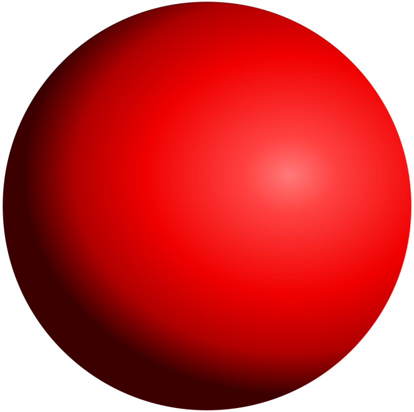
- Fuzzy spaces are (among other things) toy models of spaces with quantum structure.
- There is an interesting (new) way to describe functions and operators on fuzzy spaces.
- The description works in position space and uses the coherent states also as a basis for the functions and operators.
- Fuzzy scalar field theories are very different from their standard counterparts.
- This description allows us to see and understand this difference in position space.

String modes and **fuzzy** field theories in the string modes formalism

Quantum structure of spacetime

- We need a quantum theory of gravity.
- Quantization of general relativity leads to a nonrenormalizable theory.
- We have reasons to believe that future theory of quantum gravity will have a different notion of spacetime.
  No distinction between points under certain length scales. [Hossenfelder 1203.6191]
- Reasons:
  - gravitational Heisenberg microscope,
  - instability of quantum gravitational vacuum, [Doplicher, Fredenhagen, Roberts '95]
  - emergent spacetime.

Fuzzy spaces



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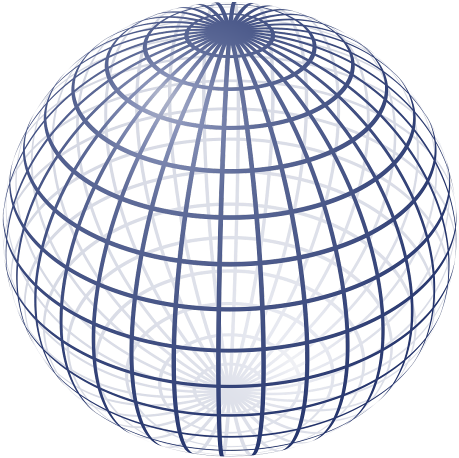
Fuzzy spaces



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Fuzzy sphere [Hoppe 1982; Madore 1992; Grosse, Klimčík, Prešnajder 1990s]

- The regular sphere S^2 is given by the coordinates

$$x_i x_i = R^2 \quad , \quad x_i x_j - x_j x_i = 0 \ , \ i = 1,2,3 \ ,$$

which generate the following algebra of functions

$$f = \left\{ \sum_{k \in \mathbb{N}^3} \left(a_{k_1 k_2 k_3} \prod_{i=1}^{3} x_i^{k_i} \right) \middle| x_i x_i = R^2 \right\} \ ,$$

which is by definition commutative.

- Information about the sphere is hidden in this algebra.

Fuzzy spaces

- For the fuzzy sphere S^2_N we define

$$\hat{x}_i\hat{x}_i = r^2 \quad , \quad \hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta\varepsilon_{ijk}\hat{x}_k \ , \ i = 1,2,3 \ .$$

- Such $\hat{x}_i$'s generate a different, non-commutative, algebra and S^2_N is an object, which has this algebra as an algebra of functions.
- The conditions can be realized as an $N = 2s+1$ dimensional representation of $SU(2)$

$$\hat{x}_i = \frac{2r}{\sqrt{N^2-1}}L_i \quad , \quad \theta = \frac{2r}{\sqrt{N^2-1}} \sim \frac{2}{N} \quad , \quad \rho^2 = \frac{4r^2}{N^2-1}s(s+1) = r^2 \ .$$

- The group $SU(2)$ still acts on $\hat{x}_i$'s and this space enjoys a full rotational symmetry.
- In the limit $N \to \infty$ we recover the original sphere.

- Most importantly nonzero commutators

$$\hat{x}_i\hat{x}_i = \rho^2 \quad , \quad \hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta\varepsilon_{ijk}\hat{x}_k \ , \ i = 1,2,3 \ .$$

imply uncertainty relations for positions

$$\Delta x_i \Delta x_j \neq 0 \ .$$

- Configuration space is analogous to phase space of quantum mechanics.
- In a similar fashion it is possible to construct an analogous deformation of the plane

$$\hat{x}_i\hat{x}_j - \hat{x}_j\hat{x}_i = i\theta\varepsilon_{ij} \ , \ i = 1,2 \ .$$

Fuzzy spaces

- $\hat{x}_i$'s are $N \times N$ matrices, functions on S_F^2 are combinations of all their possible products and thus hermitian matrices M.
- Such $N \times N$ matrix can be decomposed into

$$M = \sum_{l=0}^{N-1} \sum_{m=-l}^{m=l} c_{lm} T_{lm} \ .$$

where matrices T_{lm} are fuzzy analogues of spherical harmonics Y_{lm} (called polarization tensors)

$$\mathrm{Tr}\left(T_{lm}^{\dagger} T_{l'm'}\right) = \delta_{ll'} \delta_{mm'} \ ,$$
$$[L_i, [L_i, T_{lm}]] = l(l+1) T_{lm} \ .$$

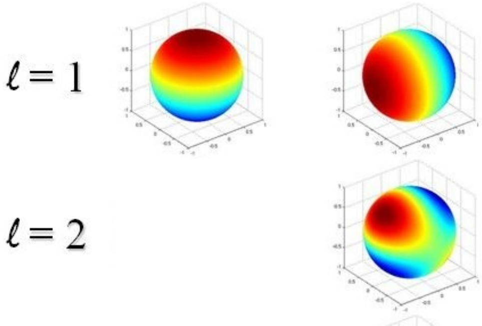
- The space of matrices M is N^2 dimensional.

Fuzzy spaces

- If we truncate the possible values of l in the expansion of functions in terms of spherical harmonics

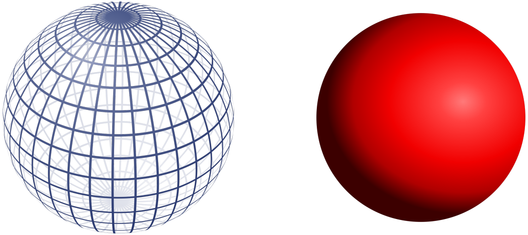
$$f = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi) \rightarrow \sum_{l=0}^{L} \sum_{m=-l}^{l} c_{lm} Y_{lm}(\theta, \phi) \ ,$$

  we will not be able to see any features of functions under certain length scales.
- Points on the sphere (as δ-functions) cease to exist.
- Expressions defined in this way are not closed under multiplication.


$\ell = 1$
$\ell = 2$

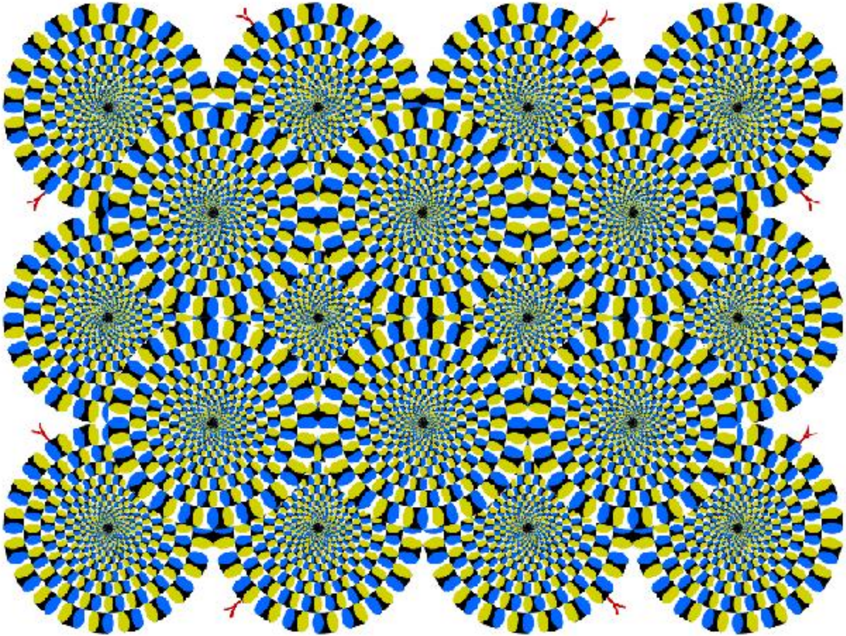
Fuzzy spaces

- We have divided the sphere into N cells. Function on the fuzzy sphere is given by a matrix M and the eigenvalues of M represent the values of the function on these cells.



An auxiliary Hilbert space has been used in the construction.

- There are no sharp boundaries between the pieces and everything is blurred, or fuzzy.



String modes and fuzzy field theories in the string modes formalism

String modes – coherent states

- Natural basis in the auxiliary hilbert space $\mathcal{H}$ is the "spin" basis

$$|n\rangle = \begin{pmatrix} \vdots \\ 1 \\ \vdots \end{pmatrix} \ , \ n = -s, \dots, s \ ,$$

  derived from the highest weight state $|s\rangle$.

- For any $x \in S^2$ with radius 1, choose some $g_x \in SO(3)$ such that $x = g_x \cdot p$, where p is the north pole on S^2. We define [Perelomov 1986]

$$|x\rangle = g_x \cdot |s\rangle \ , \ g_x \in SU(2)$$

  and call the set of all $|x\rangle$ the coherent states.

- $|x\rangle$ is located around x, but is an element of $\mathcal{H}$, and is a non-commutative analogue of the point x. [Steinacker 2020]

String modes – coherent states

- They form an over-complete set in $\mathcal{H}$ and

$$\mathbb{1} = \frac{N}{4\pi} \int d^2x \, |x\rangle \langle x| \quad , \quad \mathbb{1} = \sum_n |n\rangle \langle n| \ .$$

- They are orthogonal only in the large N limit

$$|\langle x|y\rangle|^2 = \left(\frac{1 + x \cdot y}{2} \right)^{N-1} \quad , \quad \langle m|n\rangle = \delta_{m,n} \ .$$

STRING MODES – COHERENT STATES

- Coherent states can be used to map (quantize) functions on S^2 on matrices

$$\phi(x) \to M = \int d^2x\, \phi(x) |x\rangle\langle x| \ .$$

and matrices on functions (de-quantize)

$$M \to \phi(x) = \langle x| \, M \, |x\rangle \ .$$

- This maps T_{lm} on Y_{lm} up to normalization

$$T_{lm} \to \langle x| \, T_{lm} \, |x\rangle = \frac{1}{c_l} Y_{lm}(x) \ , \ c_l^2 = \frac{1}{4\pi} \frac{(N-1-l)!(N+l)!}{((N-1)!)^2} \sim \frac{N}{4\pi} e^{\frac{l^2}{N}} \ .$$

- For $l < \sqrt{N}$ coefficients c_l are approximately constant, quantization and de-quantization are inverse of each other.
  For $l > \sqrt{N}$ coefficient c_l grows extremely fast and de-quantized matrices are misleading.

String modes – representation of functions on S_F^2

- Functions on the fuzzy sphere are matrices acting on $\mathcal{H}$

$$M = \sum_{m,n=-s}^{s} M_{mn} |m\rangle \langle n| \ .$$

- We can express the matrix M in a similar fashion using the coherent states

$$M = \left(\frac{N}{4\pi}\right)^2 \int d^2x \, d^2y \, \phi(x,y) |x\rangle \langle y| \ .$$

- Objects [Iso, Kawai, Kitazawa 2000; Steinacker 2016; Steinacker, JT 2022]

$$|x\rangle \langle y| =: \left| \begin{matrix} x \\ y \end{matrix} \right)$$

form a basis of functions on the fuzzy sphere and we will call them the **string modes**.

STRING MODES – REPRESENTATION OF FUNCTIONS ON S_F^2

- Such representation of matrix M by function $\phi(x,y)$ seems to be not unique (way more functions than matrices).
  But one can show that derivatives of $\phi(x,y)$ are bounded by $\sqrt{N}$, which means that the Fourier modes of ϕ are restricted by $l_x, l_y \leq \sqrt{N}$.
- Functions of two variables $\phi(x,y)$ that represent functions on the fuzzy sphere have rather mild behavior.
  The coherent states are spread out over an area $\sim 4\pi/N$ and average out any larger oscillations.
- Large momentum UV wavelengths are smoothed out on the fuzzy sphere. But the price we pay is non-local string modes.

String modes – representation of functions on S^2_F

$$\left|\begin{matrix} x \\ y \end{matrix}\right)$$

- **Short modes** for $|x-y| < 1/\sqrt{N}$ can be shown to represent localized wave-packets with momentum $\sim N|x-y|$.
  This is the classical regime.
- Particularly string mode $\left|\begin{matrix} x \\ x \end{matrix}\right)$ represents a maximal localized function around point x, i.e. a fuzzy version of δ-function.
  Functions with $\phi(x,y) = \phi(x)\delta(x,y)$ are local and become the standard functions on S^2 in the commutative limit.
- **Long modes** for $|x-y| > 1/\sqrt{N}$ are non-local and have no classical analogue.
  This is the non-commutative regime.

String modes – representation of operators on functions

- When working with functions we encounter operators

$$\mathcal{O} : M \to \mathcal{O}(M) \ .$$

- For example the Laplace opeartor, the kinetic term of the field theory, or the propagator

$$[L_i, [L_i, M]] =: \Box M \quad , \quad \frac{1}{\Box + m^2} \ .$$

- String modes are (large N) eigenfunctions of $\Box$

$$\Box \left| {x \atop y} \right) \approx \left(\frac{N^2}{4} |x - y|^2 + N \right) \left| {x \atop y} \right) \ .$$

[Steinacker, JT 2022]

- A general representation of such operators in terms of the string modes is straightforward

$$\mathcal{O} = \left(\frac{N}{4\pi}\right)^4 \int d^2x\, d^2x'\, d^2y\, d^2y' \left|{x \atop y}\right) \mathcal{O}(x,y;x',y') \left({x' \atop y'}\right| .$$

- There are two special cases
  - Local

$$\mathcal{O} = \left(\frac{N}{4\pi}\right)^2 \int d^2x\, d^2y \left|{x \atop x}\right) \mathcal{O}^L(x,y) \left({y \atop y}\right| .$$

  - Non-local, but diagonal,

$$\mathcal{O} = \left(\frac{N}{4\pi}\right)^2 \int d^2x\, d^2y \left|{x \atop y}\right) \mathcal{O}^D(x,y) \left({x \atop y}\right| .$$

- Functions $\mathcal{O}_L$ and $\mathcal{O}_D$ may have very different behavior for different operators (oscillation, singularity). Local representations are typically highly oscillatory, non-local representations are better behaved.

- For the propagator

$$\frac{1}{\Box + m^2} = \left(\frac{N}{4\pi}\right)^2 \int d^2x\, d^2y \left|{x \atop y}\right) \mathcal{O}_P^D(x,y) \left({x \atop y}\right|$$

where

$$\mathcal{O}_P^D(x,y) = \left({x \atop y}\right| \frac{1}{\Box + m^2} \left|{x \atop y}\right) \ .$$

- Operator traces

$$\mathrm{Tr}\, \mathcal{O} = \left(\frac{N}{4\pi}\right)^2 \int d^2x\, d^2y \left({x \atop y}\right| \mathcal{O} \left|{x \atop y}\right) \ .$$

- For any function of the $\Box$ operator $f(\Box)$ we have

$$\left({x \atop x} \middle| f(\Box) \middle| {y \atop y} \right) = \frac{1}{N} \sum_l (2l+1) f\left(l(l+1)\right) e^{-l^2/N} P_l(\cos\vartheta)$$

$$\left({x \atop y} \middle| f(\Box) \middle| {x \atop y} \right) = \frac{1}{N} \sum_{k,l} (2k+1)(2l+1)(-1)^{l+k+2s} f\left(k(k+1)\right) \begin{Bmatrix} l & s & s \\ k & s & s \end{Bmatrix} e^{-l^2/N} P_l(\cos\vartheta)$$

where the curly bracket is the $6j$-symbol and $\cos\vartheta = x \cdot y$.

- For the propagator we obtain

$$\left({x \atop y} \middle| \frac{1}{\Box + m^2} \middle| {x \atop y} \right) \approx \frac{1}{\frac{N^2}{4}|x-y|^2 + m^2} \ .$$

String modes – representation of operators on functions

- Trace of propagator

$$\mathrm{Tr}\,\frac{1}{\Box+m^2}=\frac{N^2}{(4\pi)^2}\int d^2x\,d^2y\,\left(\begin{smallmatrix}x\\y\end{smallmatrix}\right|\frac{1}{\Box+m^2}\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)=\frac{N^2}{(4\pi)^2}\int d^2x\,d^2y\,\frac{1}{\frac{N^2}{4}|x-y|^2+m^2}=$$

$$=\frac{N^2}{2}\int_{-1}^{1}du\,\frac{1}{\frac{N^2}{2}(1-u)+m^2}\sim 2\log(N)\ .$$

- This is consistent with

$$\mathrm{Tr}\,\frac{1}{\Box+m^2}=\sum_{l=0}^{N-1}\frac{2l+1}{l(l+1)+m^2}\sim N\int_0^1\frac{2Nx}{N^2x^2+m^2}\sim 2\log(N)\ .$$

String modes and fuzzy field theories in the string modes formalism

Fuzzy scalar field theory

- **Commutative** (for S^2)

$$S(\Phi) = \int d^2x \left[\frac{1}{2}\Phi\Delta\Phi + \frac{1}{2}m^2\Phi^2 + V(\Phi)\right]$$

$$\langle F \rangle = \frac{\int d\Phi\, F(\Phi) e^{-S(\Phi)}}{\int d\Phi\, e^{-S(\Phi)}} .$$

- **Non-commutative** (for S_F^2)

$$S(M) = \frac{4\pi r^2}{N} \mathrm{tr} \left[\frac{1}{2} M \frac{1}{r^2}[L_i,[L_i,M]] + \frac{1}{2}m^2 M^2 + V(M)\right]$$

$$\langle F \rangle = \frac{\int dM\, F(M) e^{-S(M)}}{\int dM\, e^{-S(M)}} .$$

[Balachandran, Kürkçüoğlu, Vaidya 2005; Szabo 2003; Ydri 2016]

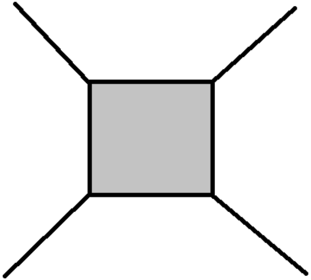
Fuzzy scalar field theory – UV/IR mixing

- The key property of the non-commutative field theories is the UV/IR mixing phenomenon, which arises as a result of the nonlocality of the theory.
  [Minwalla, Van Raamsdonk, Seiberg '00; Chu, Madore, Steinacker '01]
- Very energetic fluctuations (UV physics) have consequences at large distances (IR physics).
- In terms of diagrams different properties of planar and non-planar ones.
  The (matrix) vertex is not invariant under permutation of incoming momenta.

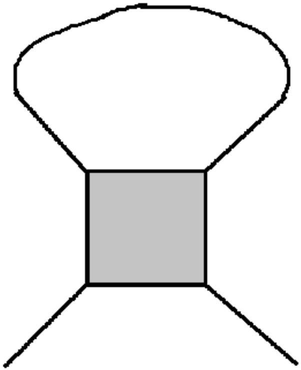
$$M = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} c_{lm} T_{lm} \ , \ \ S(M) = \frac{4\pi}{N} \mathrm{tr} \left[\frac{1}{2} M[L_i,[L_i,M]] + \frac{1}{2} m^2 M^2 + g\, M^4 \right]$$

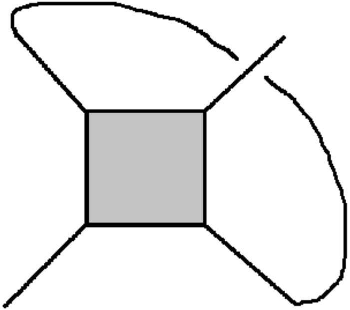
Fuzzy scalar field theory – UV/IR mixing

$$\mathrm{Tr}\left(M^4\right) = \sum_{l_{1\ldots 4}} \sum_{m_{1\ldots 4}} c_{l_1 m_1} c_{l_2 m_2} c_{l_3 m_3} c_{l_4 m_4} \mathrm{tr}\left(T_{l_1 m_1} T_{l_2 m_2} T_{l_3 m_3} T_{l_4 m_4}\right)$$



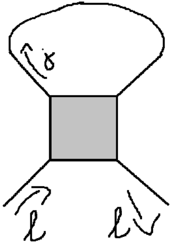
Fuzzy scalar field theory - UV/IR mixing

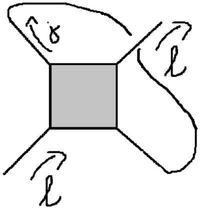




Fuzzy scalar field theory – UV/IR mixing

[Chu, Madore, Steinacker '01]





$$I^{P} = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2}$$

$$I^{NP} = \sum_{j=0}^{N-1} \frac{2j+1}{j(j+1)+m^2} N(-1)^{l+j+N-1} \begin{Bmatrix} l & s & s \\ j & s & s \end{Bmatrix}, \quad s = \frac{N-1}{2}$$

Fuzzy scalar field theory – UV/IR mixing

- $N \to \infty$ limit of the effective action is different from the standard S^2 effective action

$$S_{\text{one loop}} = S_0 + \frac{1}{2}\int d^2x\,\phi(x)^2\delta m^2 - \frac{g}{12\pi}\int d^2x\,\phi(x)h(\tilde{\Delta})\phi(x) + \dots$$

$$\tilde{\Delta}Y_{lm} = l\,Y_{lm}\ ,\ \ h(n) = \sum_{k=1}^{n}\frac{1}{k}\ .$$

There is an extra, mildly nonlocal, term.

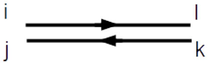
- Expressing M in terms of T_{lm} diagonalizes the kinetic term and leaves us to struggle with the interaction term.
- There is a different treatment that is more favorable to the interaction term but the kinetic term is the problematic one now.

Fuzzy scalar field theory – matrix formalism

- If we forget about the kinetic term for a moment

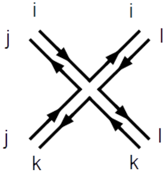
$$S(M) = S_{kinetic} + \frac{4\pi}{N}\left[\frac{1}{2}m^2 M_{ij}M_{ji} + g\, M_{ij}M_{jk}M_{kl}M_{li}\right]$$

we can treat this model as a field theory with propagator



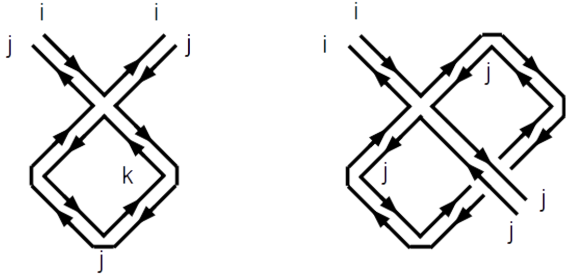
$$= \langle M_{ij}M_{kl}\rangle \sim \frac{1}{m^2}\delta_{il}\delta_{jk}$$

and vertex



$$\sim g\ .$$

- Resulting graphs are called fat graphs or ribbon graphs and are well known in matrix models.



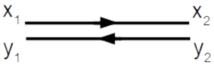
- Matrix indexes run in the loops, not the momenta!
- The problem is that the kinetic term leads to a nondiagonal propagator.

String modes and fuzzy field theories in the string modes formalism

Loop computations and (non)locality in fuzzy QFT

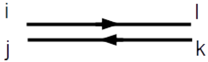
[Steinacker 2016; Steinacker, JT 2022]

- Feynman rules in string modes formalism - propagator



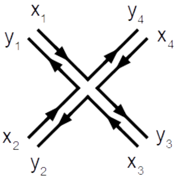
$$= \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \left| \frac{1}{\Box + m^2} \right| \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \approx \frac{1}{\frac{N^2}{4}|x-y|^2 + m^2} \langle x_2 | x_1 \rangle \langle y_2 | y_1 \rangle$$

Compare with the pure matrix models propagator



$$\sim \frac{1}{m^2} \delta_{il} \delta_{jk} \ .$$

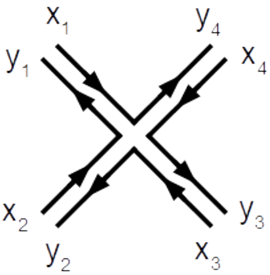
- Feynman rules in string modes formalism - vertex

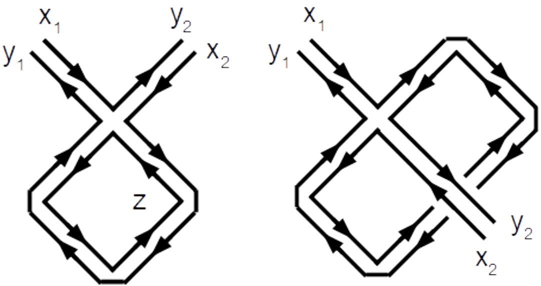


$$\sim g \langle y_1 | x_2 \rangle \langle y_2 | x_3 \rangle \langle y_3 | x_4 \rangle \langle y_4 | x_1 \rangle \ .$$

- String modes bring, in the large N limit, the best from the two worlds. They diagonalize the kinetic term and keep a simple structure of the vertices.
- Similar to the standard QFT calculations, but regular thanks to the effective non-commutative cutoff. No singularities and no issues when computing loop diagrams in position space.
- Too good to be true? The price we pay are the non-local modes $\left| {x \atop y} \right)$ running in the diagrams or a non-local theory of field $\phi(x, y)$.

One-loop two-point function and effective action





$$\frac{g}{3}\left(\frac{N}{4\pi}\right)^4 \int d^2x_1 d^2y_1 d^2x_2 d^2y_2\, \phi(x_1,y_1)\phi(x_2,y_2)\times$$
$$\times\left[\langle y_2|x_1\rangle\langle y_1|x_2\rangle\frac{m_N^2}{N}+\frac{1}{2}\langle y_2|x_2\rangle\langle y_1|x_1\rangle\frac{1}{\frac{N^2}{4}|x_1-y_2|^2+m^2}\right].$$

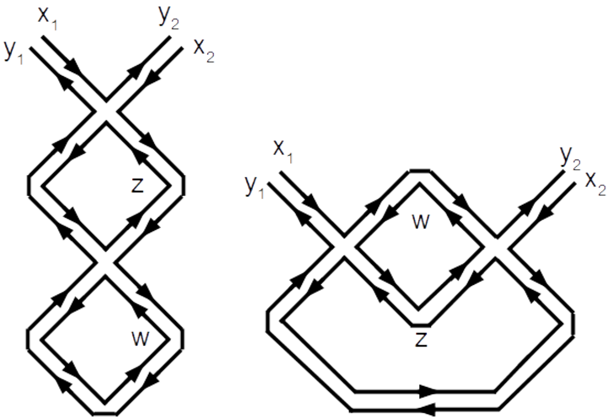
One-loop two-point function and effective action

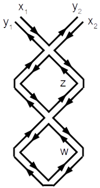
- We obtain the one-loop effective action for the classical fields $\phi(x,y)=\phi(x)\langle x|y\rangle$

$$S_{\text{eff}}=\int d^2x\,\phi(x)\frac{1}{2}(\Box+m^2)\phi(x)+g\int d^2x\,\phi(x)^4+\frac{g}{3}\frac{1}{4\pi}\int d^2x\,\phi(x)^2\mu_N^2+$$
$$+\frac{g}{6}\left(\frac{N}{4\pi}\right)^2\int d^2x\,d^2y\,\phi(x)\phi(y)\frac{1}{\frac{N^2}{4}|x-y|^2+m^2}\;.$$

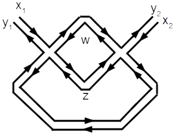
- It can be shown that this is equivalent to the previous formula with $-\frac{g}{12\pi}\int d^2x\,\phi h(\tilde{\Delta})\phi$ but with a different interpretation. [Steinacker 2016]

Two-loop two-point function and effective action





$$\approx \langle y_2|x_1\rangle \langle y_1|x_2\rangle \left(\frac{g}{4!}\right)^2 \frac{N}{4\pi}\left[\frac{N}{4\pi}\int dw\, \mathcal{O}_P(\cos\vartheta_{wy_1})\right]\left[\frac{N}{4\pi}\int dz\, \mathcal{O}_P(\cos\vartheta_{zy_1})^2\right]$$



$$\approx \langle y_2|x_1\rangle \langle y_1|x_2\rangle \left(\frac{g}{4!}\right)^2 \frac{N}{4\pi}\left[\left(\frac{N}{4\pi}\right)^2\int dzdw\, \mathcal{O}_P(\cos\vartheta_{x_1 w})\mathcal{O}_P(\cos\vartheta_{wz})\mathcal{O}_P(\cos\vartheta_{zy_1})\right]$$

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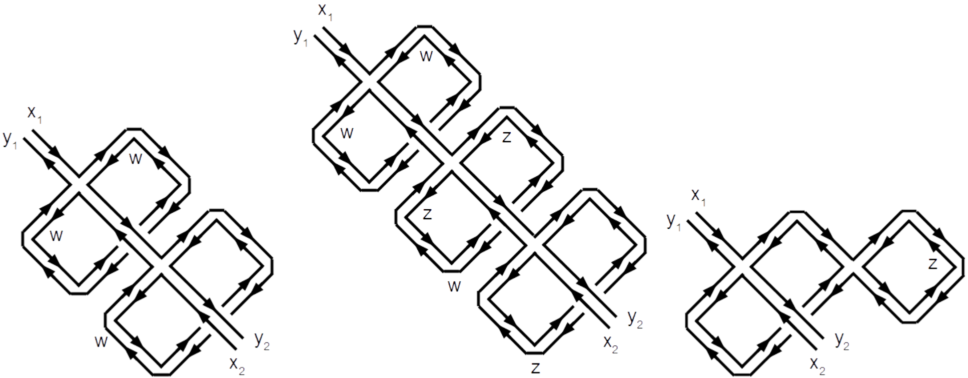
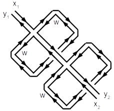
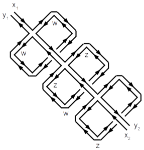


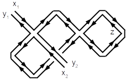


$$\approx \langle y_1|x_1\rangle\,\langle x_2|y_2\rangle \left(\frac{g}{4!}\right)^2 \frac{N}{4\pi}\left[\frac{N}{4\pi}\int dw\, \mathcal{O}_P(\cos\vartheta_{x_1 w})\mathcal{O}_P(\cos\vartheta_{wy_1})\right]\mathcal{O}_P(1)$$



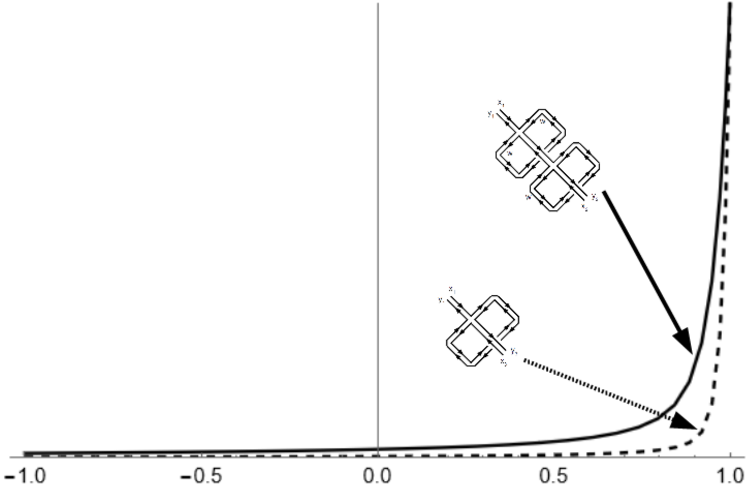
$$\approx \langle y_1|x_1\rangle\,\langle x_2|y_2\rangle \left(\frac{g}{4!}\right)^3 \left(\frac{N}{4\pi}\right)^2 \mathcal{O}_P(1)^2\times$$

$$\times\left[\left(\frac{N}{4\pi}\right)^2\int dzdw\, \mathcal{O}_P(\cos\vartheta_{x_1 w})\mathcal{O}_P(\cos\vartheta_{wz})\mathcal{O}_P(\cos\vartheta_{zy_1})\right]$$



$$\approx \langle y_1|x_1\rangle\,\langle x_2|y_2\rangle \left(\frac{g}{4!}\right)^2 \frac{N}{4\pi}\left[\frac{N}{4\pi}\int dz\, \mathcal{O}_P(\cos\vartheta_{zx_1})\right]\mathcal{O}_P(\cos\vartheta_{x_1 y_1})^2$$

Two-loop two-point function and effective action



Take home message

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- Fuzzy spaces are (among other things) toy models of spaces with quantum structure.
- Fuzzy scalar field theories are very different from their standard counterparts.
- There is an interesting (new) way to describe functions and operators on fuzzy spaces.
- The description works in position space and uses the coherent states also as a basis for the functions and operators.
- It leads to some new insights in the fuzzy scalar field theory.

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Thank you for your attention!