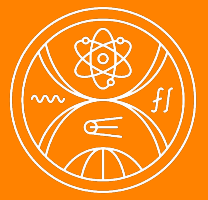
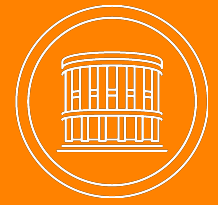


FACULTY OF MATHEMATICS,
PHYSICS AND INFORMATICS
Comenius University
Bratislava

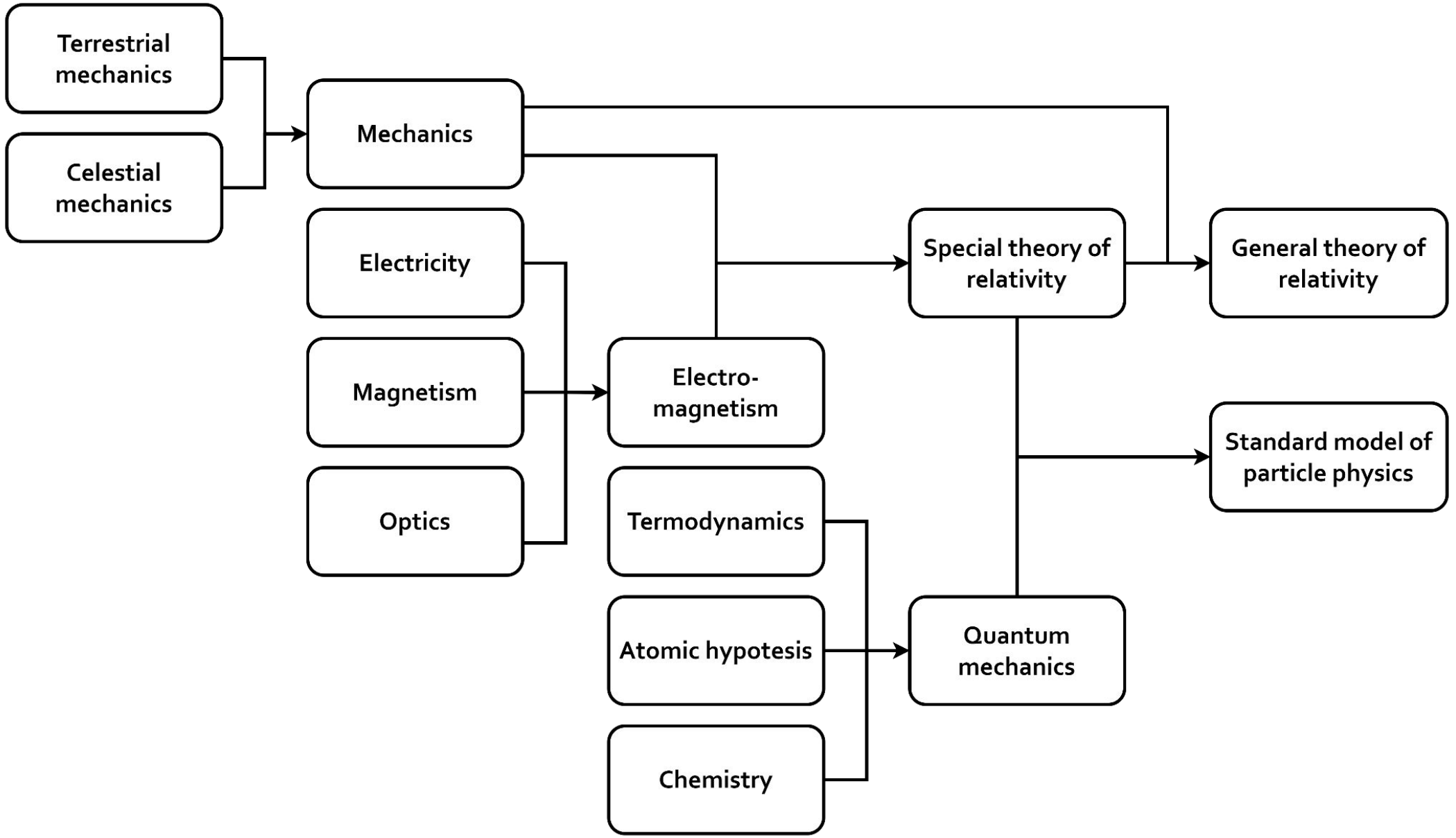
Quantum structure of spacetime

Juraj Tekel
Department of theoretical physics



4. 4. 2025, habilitation lecture, FMFI UK, Bratislava

History of physics


Terrestrial mechanics
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Mechanics
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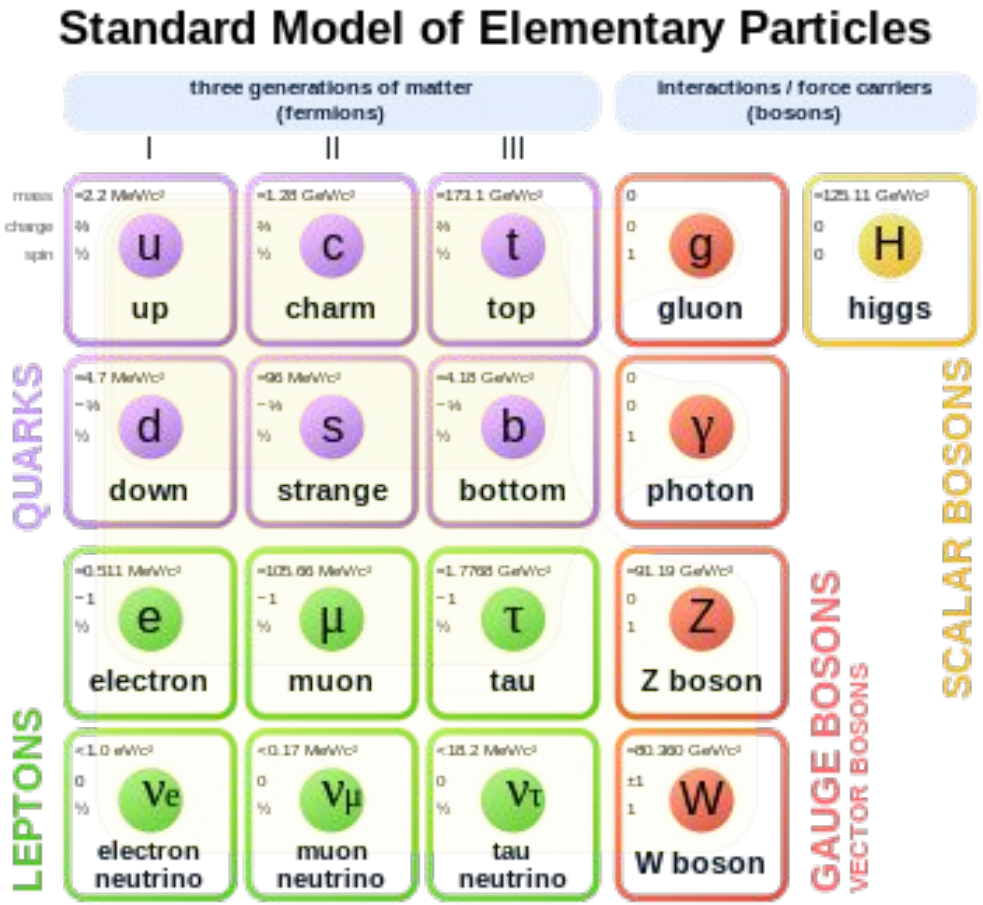
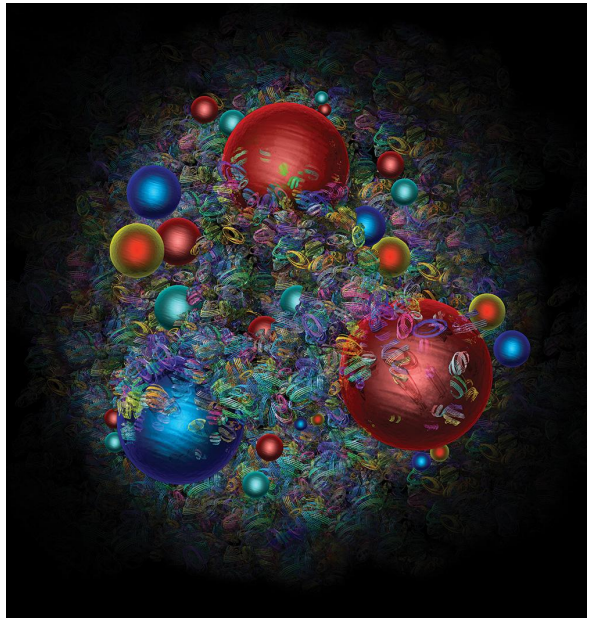
Termodynamics
Atomic hypotesis
Chemistry
Special theory of relativity
Quantum mechanics
General theory of relativity
Standard model of particle physics

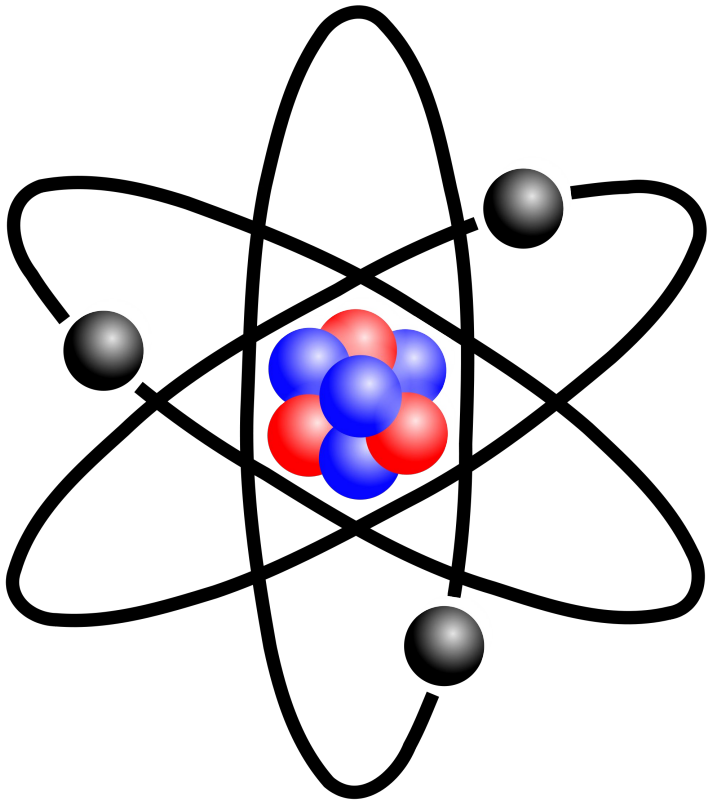
Quantum theory

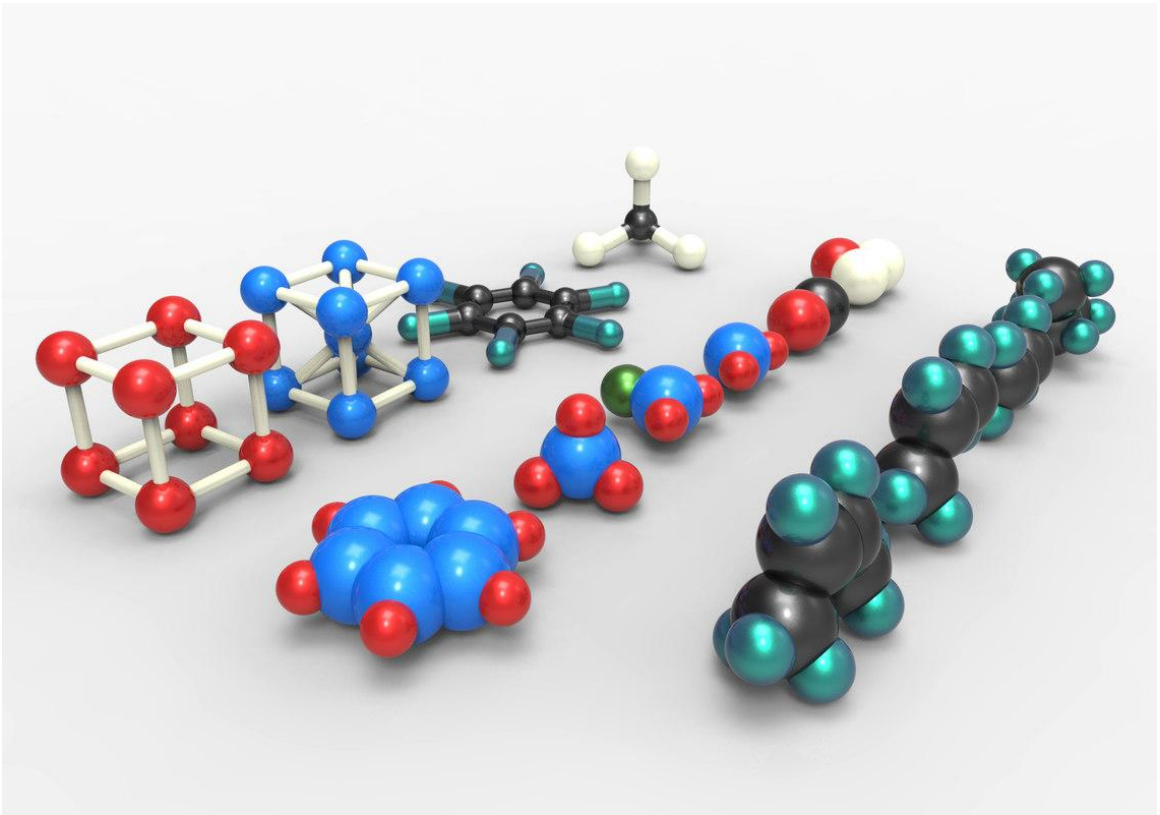




img.: wiki commons; D Dominguez/CERN

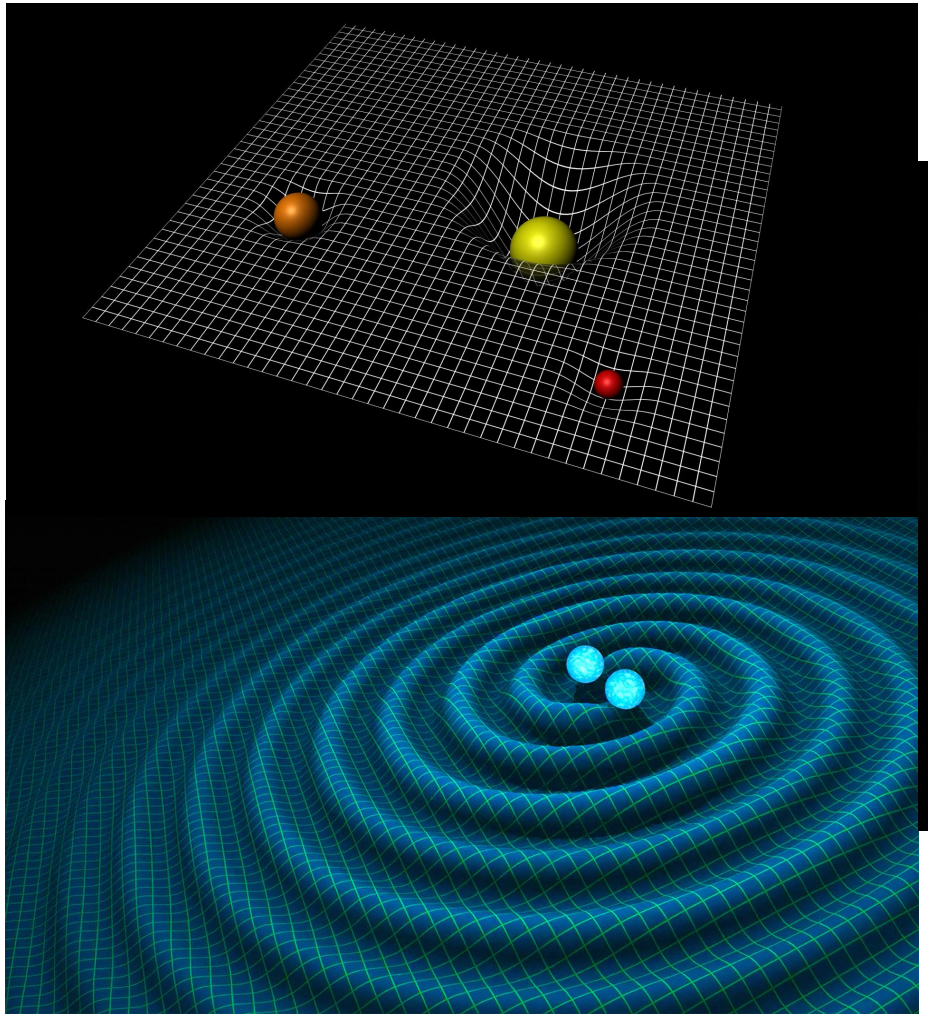
Quantum theory

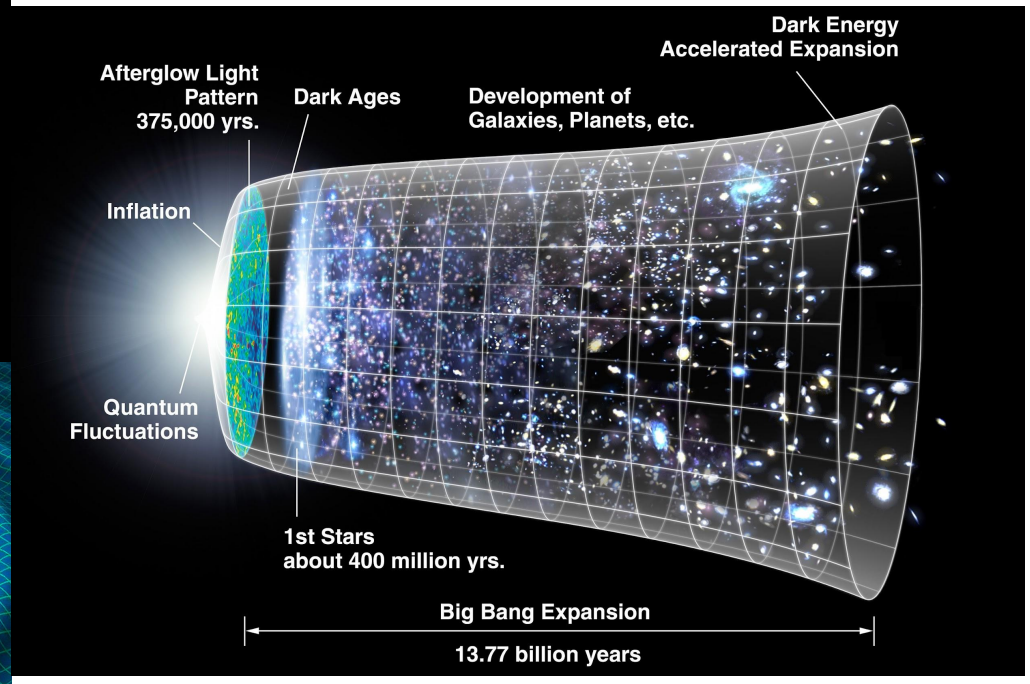




img.: Halfdan, wiki commons; cgtrader.com

General relativity





obr.: ESA–C.Carreau; R. HURT/CALTECH-JPL; NASA

General relativity

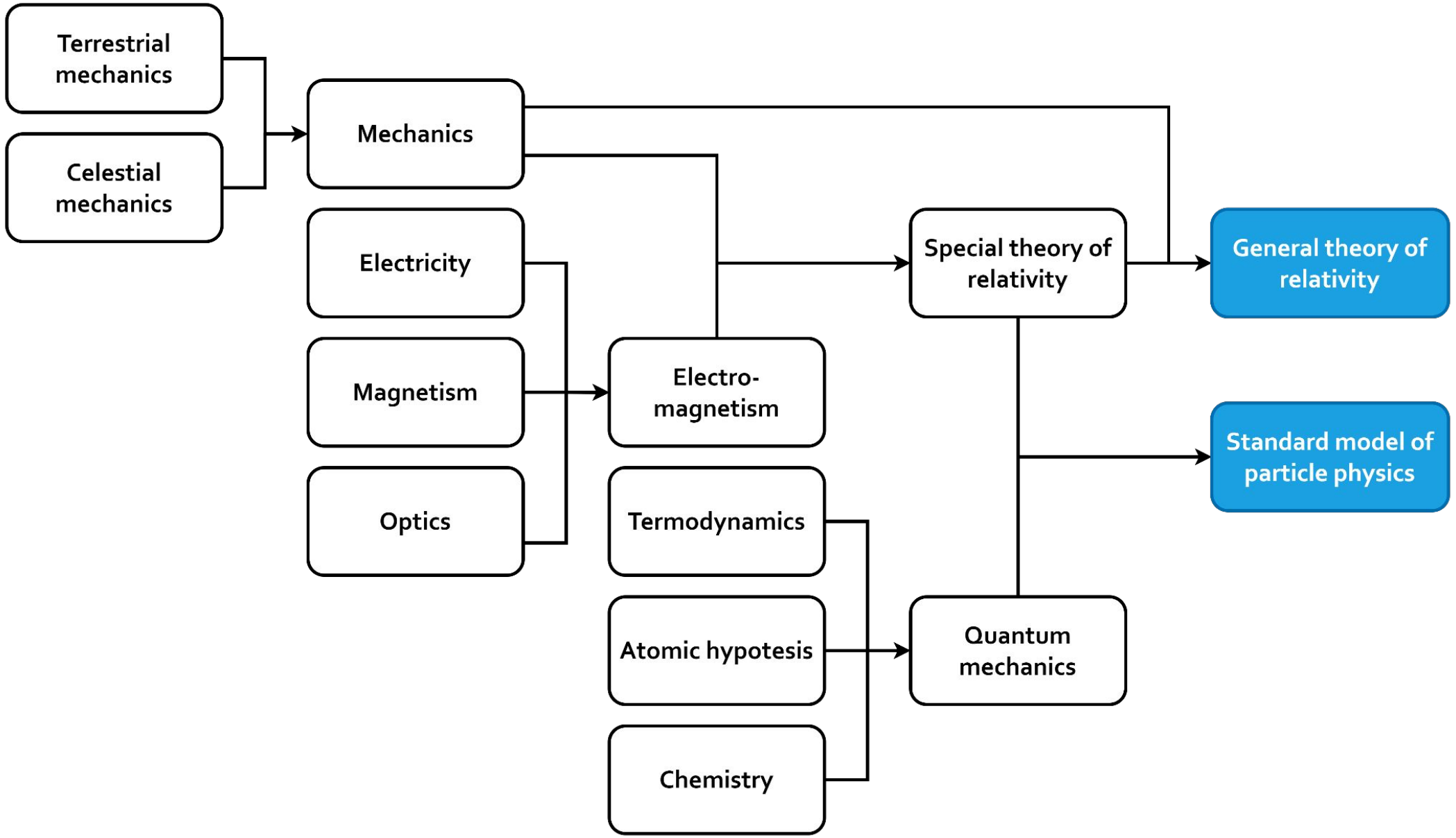
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

General relativity

matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

space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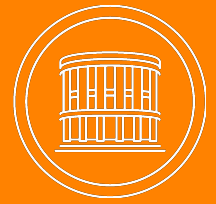
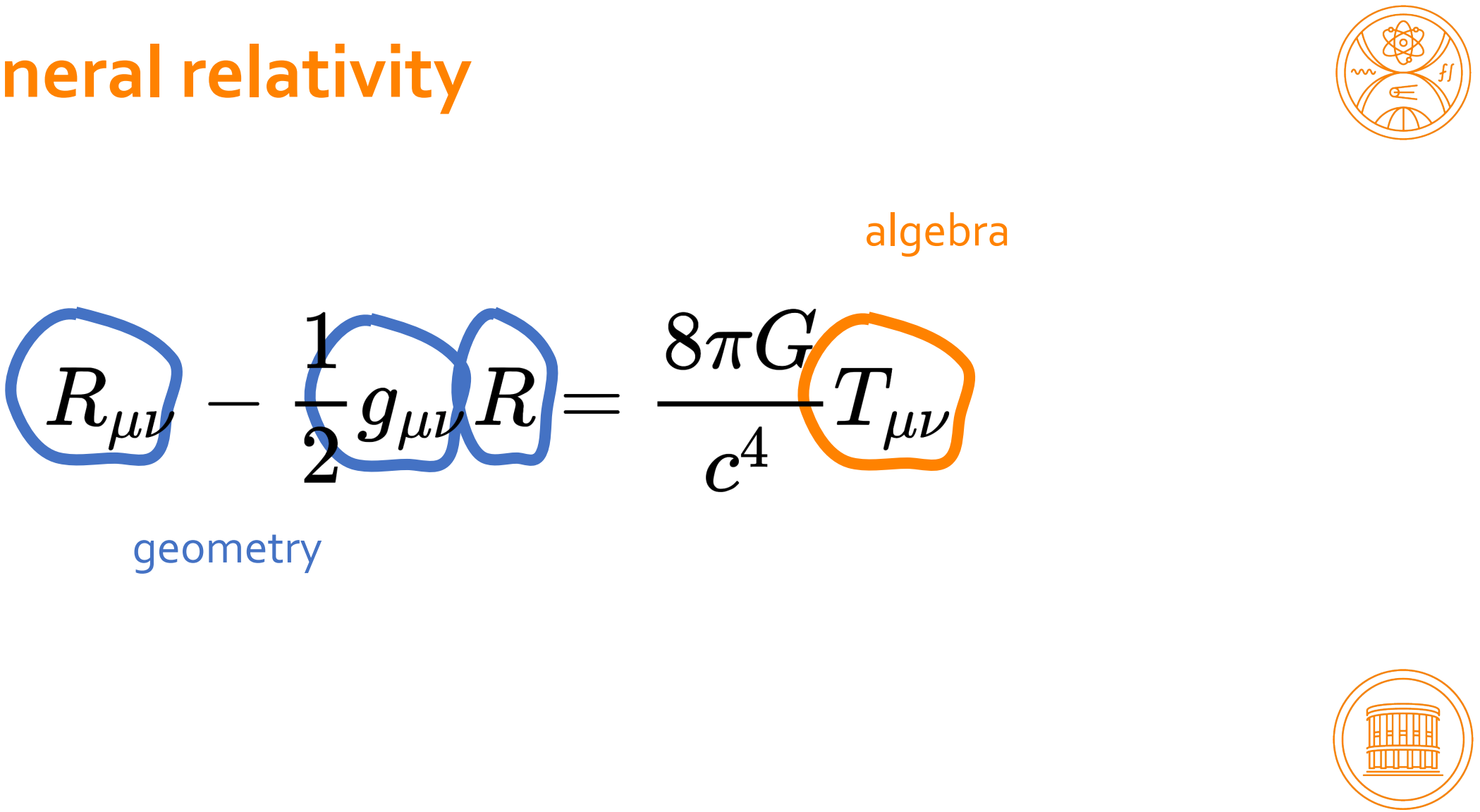
Quantum theory of gravity



? ? ?

General relativity



algebra

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

geometry

Unification of **quantum** mechanics and **gravitation** predicts structure of spacetime.

Spacetime is composed of something.

What does “being composed of something” mean?

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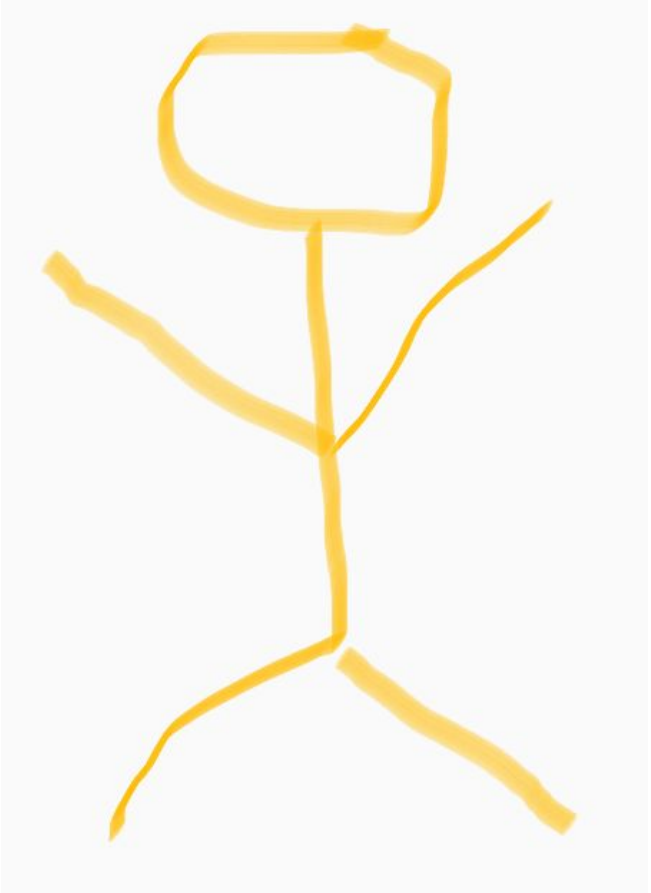
- **A person** is composed of head, arms, legs and torso.



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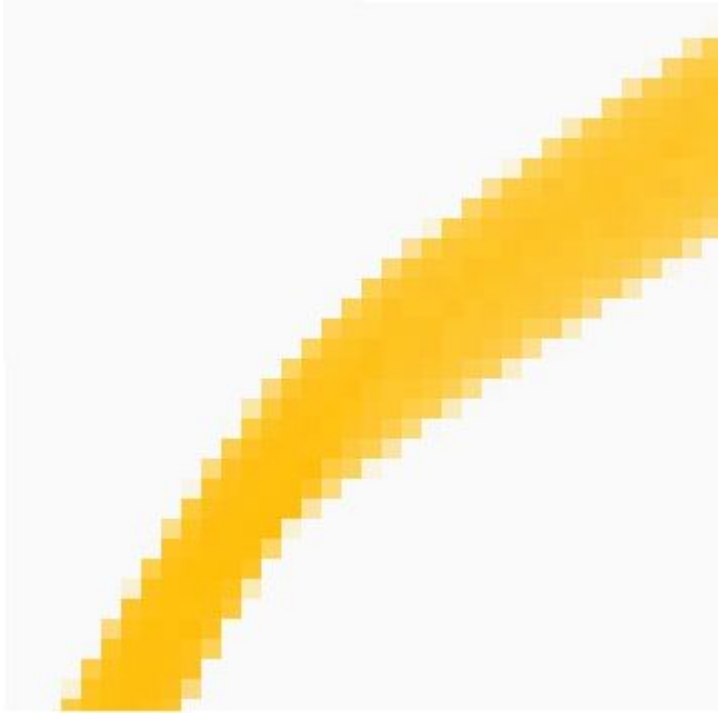

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- Everything is composed of **atoms**.

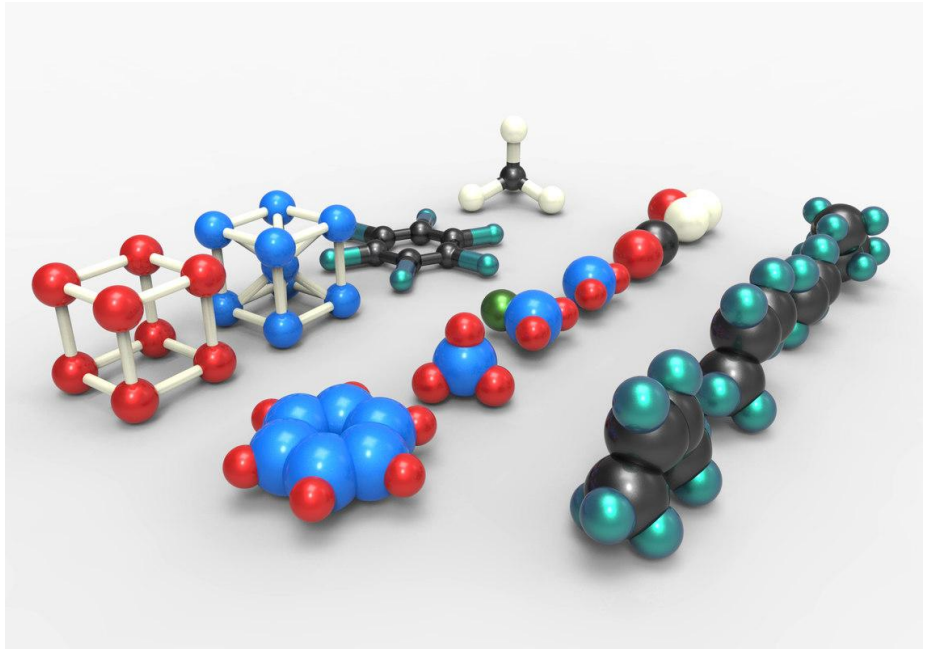

fig.: www.cgtrader.com

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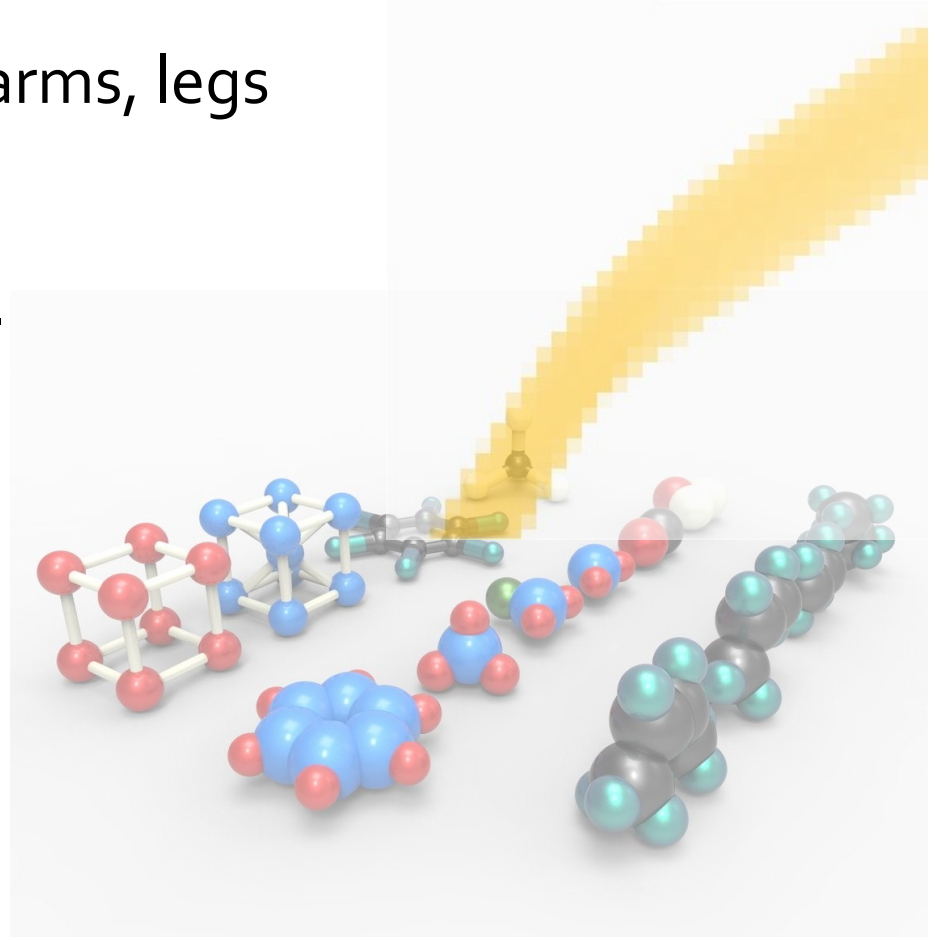

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What does "being composed of something" mean?

- **Atoms** are composed of electrons, protons and neutrons.

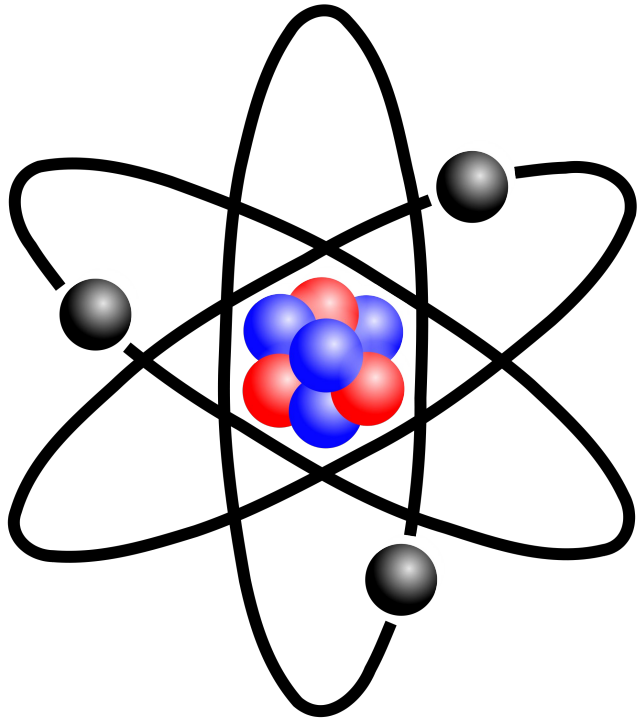
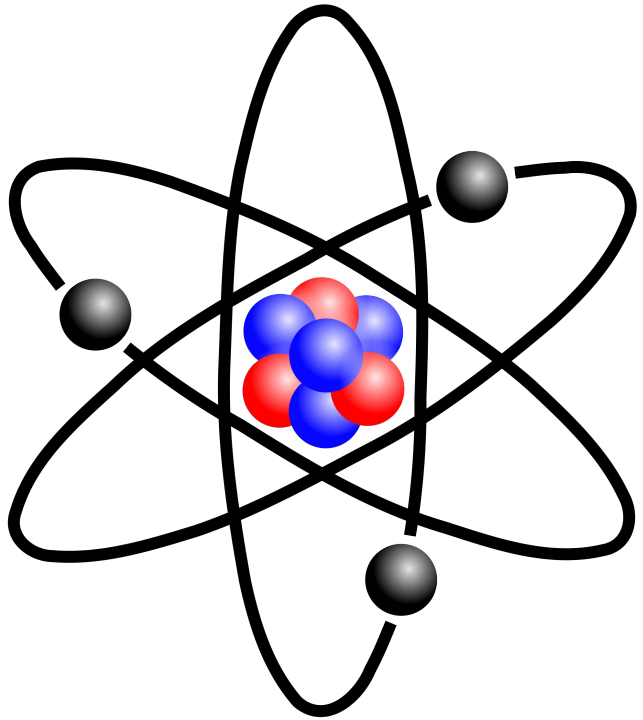


fig.: wiki, Halfdan

What does "being composed of something" mean?

- **Atoms** are composed of electrons, protons and neutrons.
- Differently, because of **quantum mechanics**.



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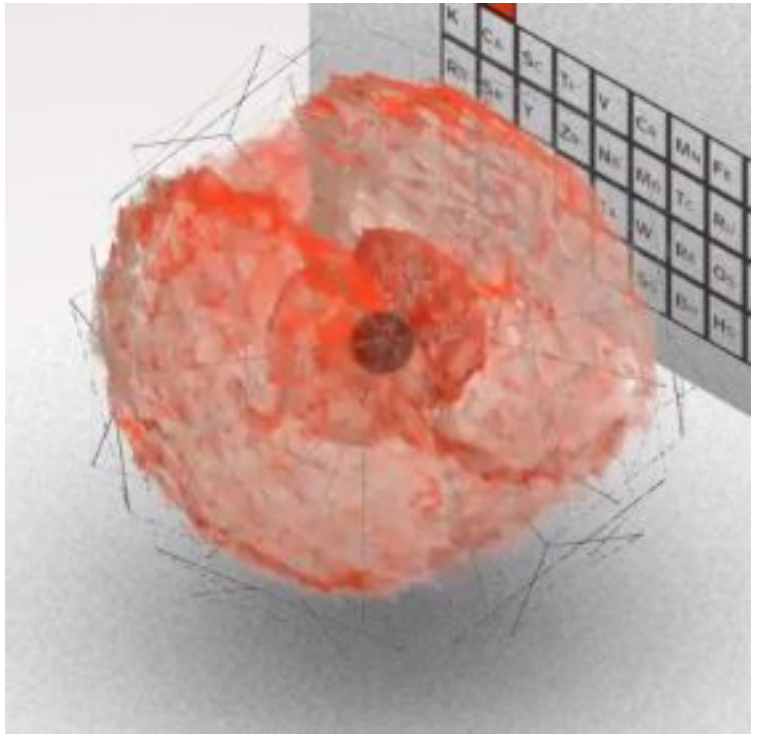

At the level of (truly) elementary objects, “to be composed of” means something different.

How large are atoms?

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- Dimension of an **atom** is roughly 10^{-10} m.
- That is very small number.

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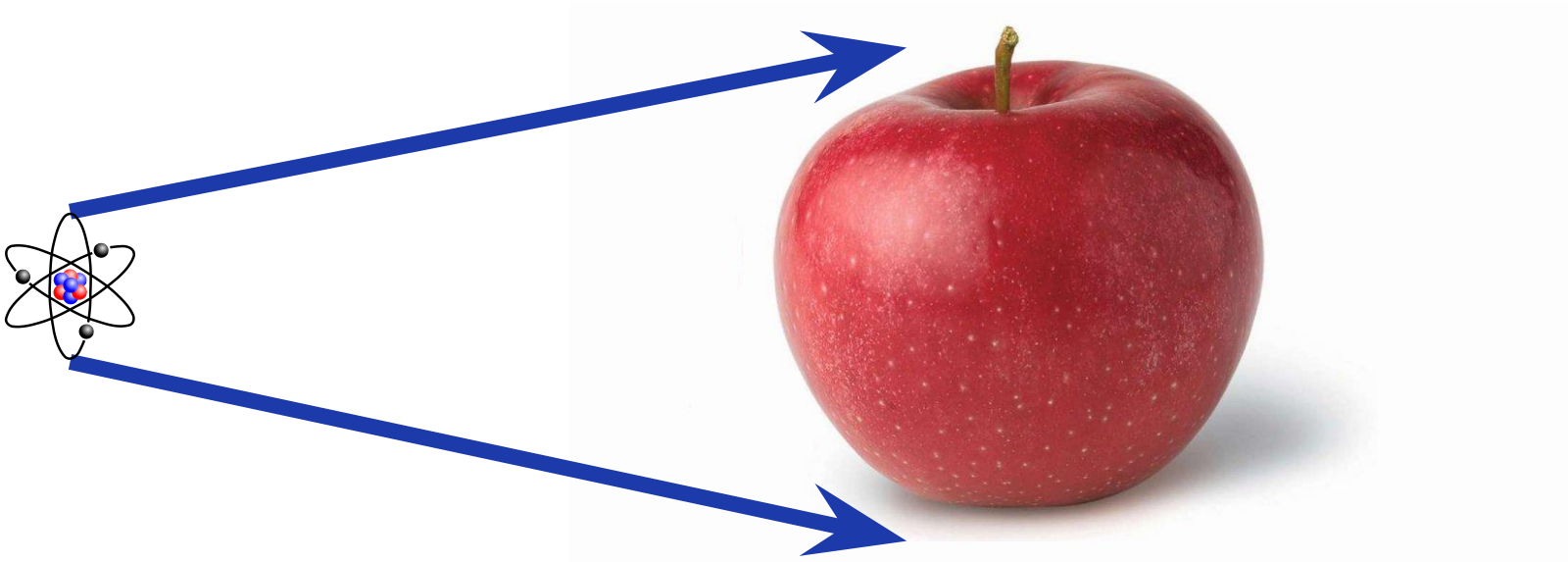

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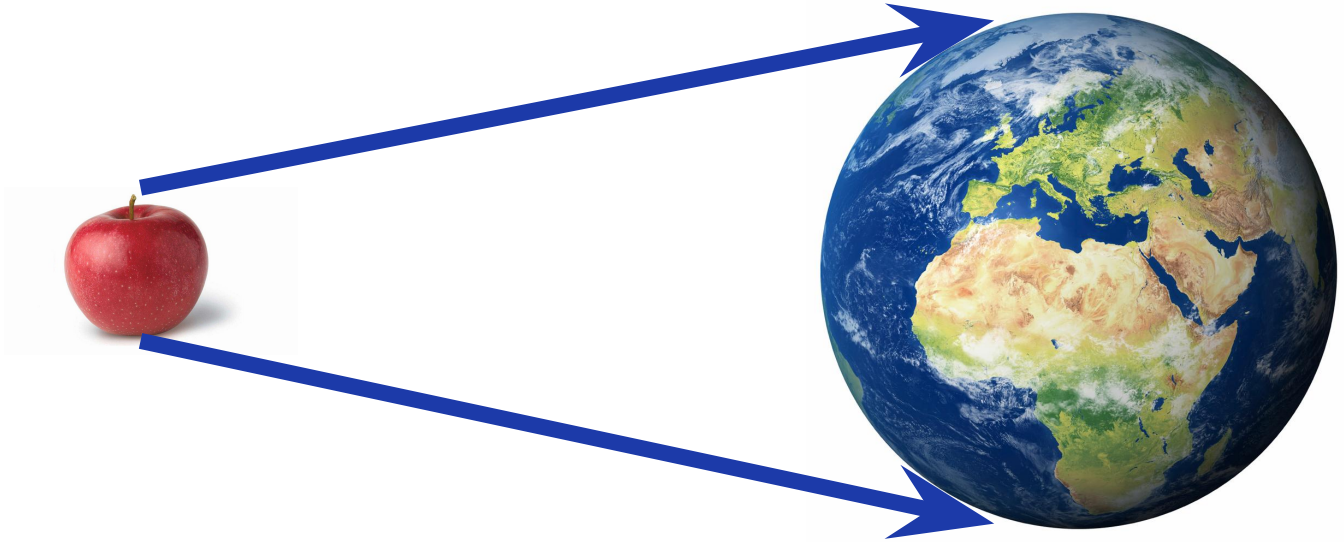

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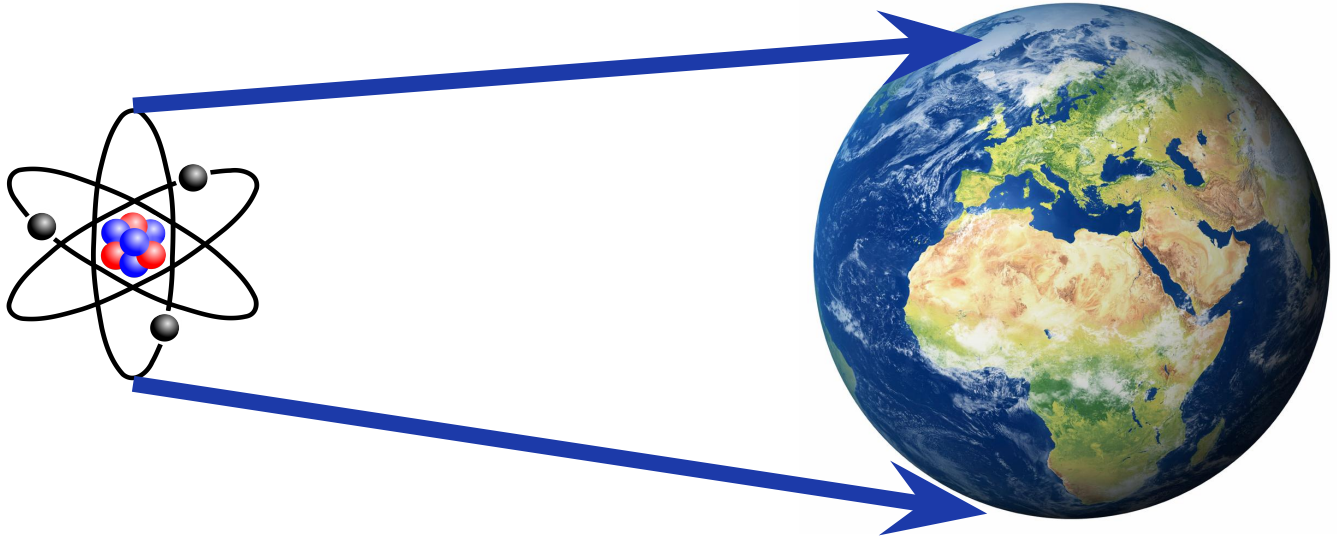

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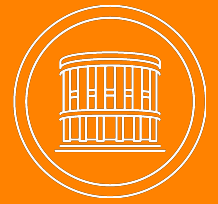
fig.: wiki

How large are atoms?

- Dimension of an **atom** is roughly 10^{-10} m.
- That is very small number.
- Dimension of a **nucleus** is roughly 10^{-15} m.
- **Meter**, **kilogram** and **second** are tailor made for humans.
- Things large / small in these units **are such for humans**.

Different situations have different suitable units.

Natural units of nature



Natural units of nature

- We seek fundamental theory, which incorporates:
  - quantum mechanics
  - theory of relativity
  - gravity

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  - Planck time 10^{-43} s
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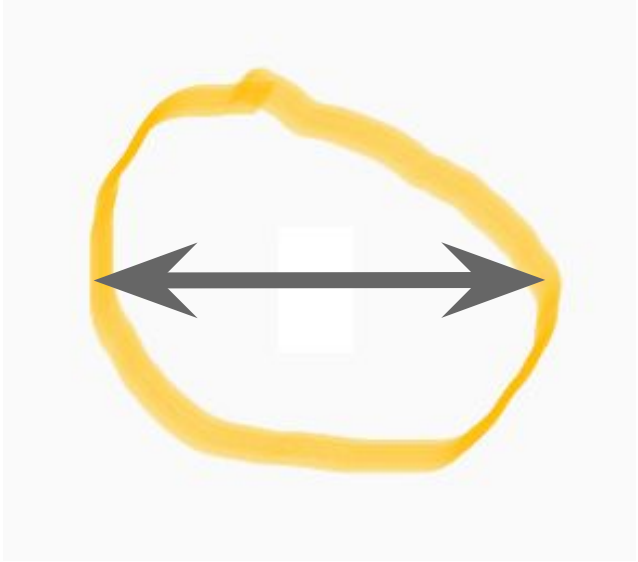
Natural units of nature

- We can combine $\hbar$, c and G to obtain:
  - Planck length 10^{-35} m
  - Planck time 10^{-43} s
  - Planck density 10^{97} kg/m^3
- These are the natural units of nature. At these scales effects of all three fundamental theories are important.

Small particles and black holes

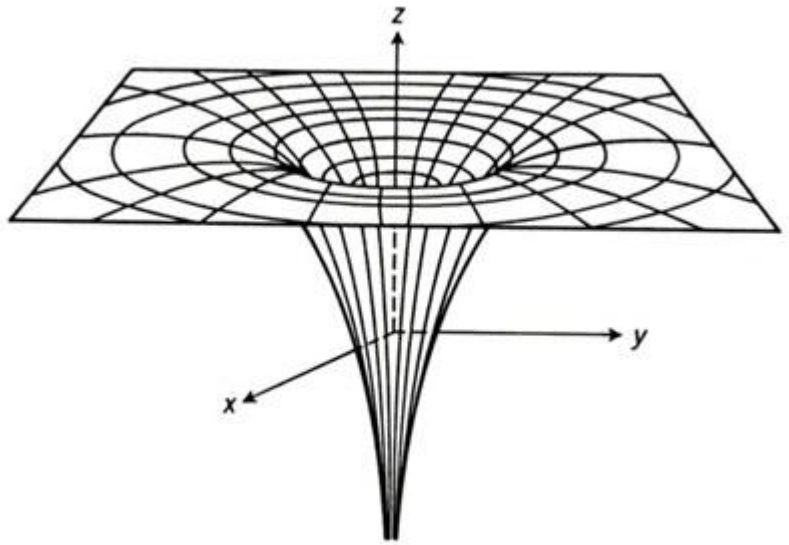
Quantum gravity

- ħ – the smaller the particle size, the large its energy



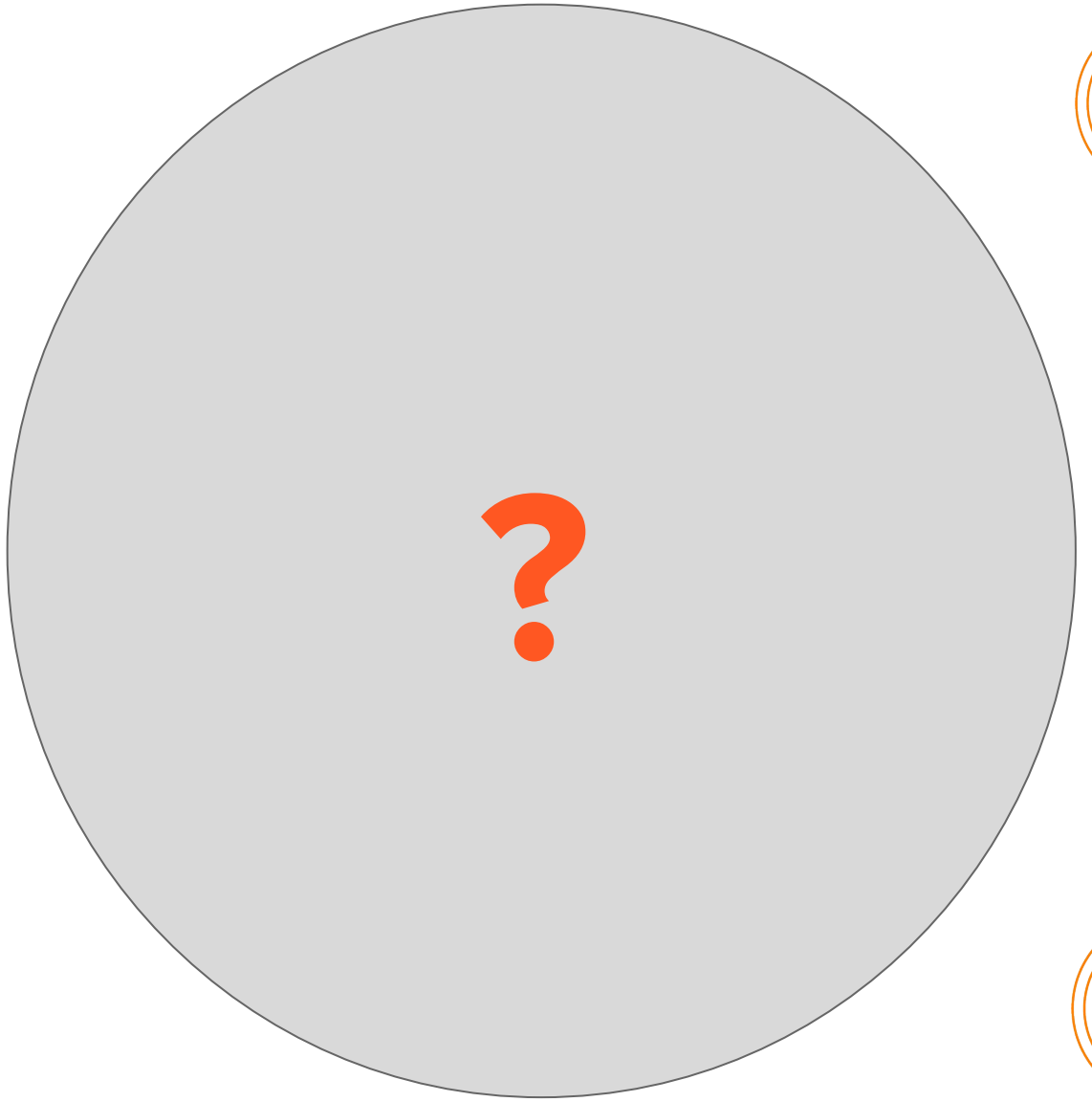
Quantum gravity

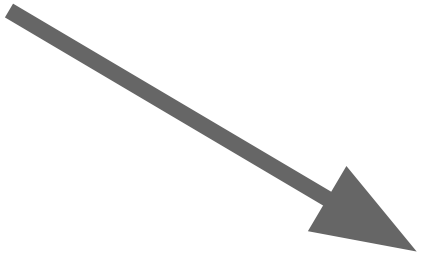
- $\hbar$ – the smaller the particle size, the large its energy
- **G** – too much energy at one place creates a black hole

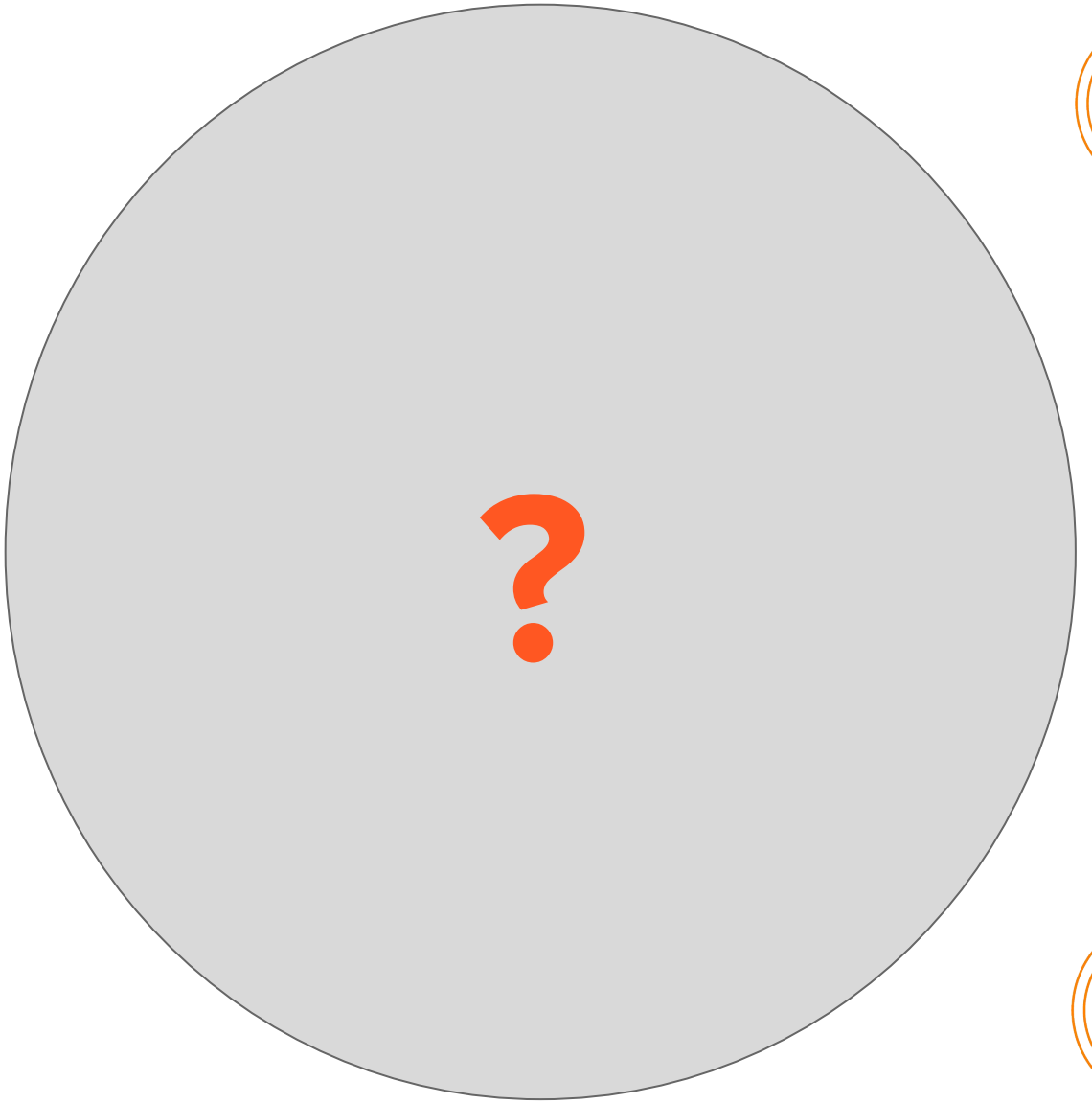


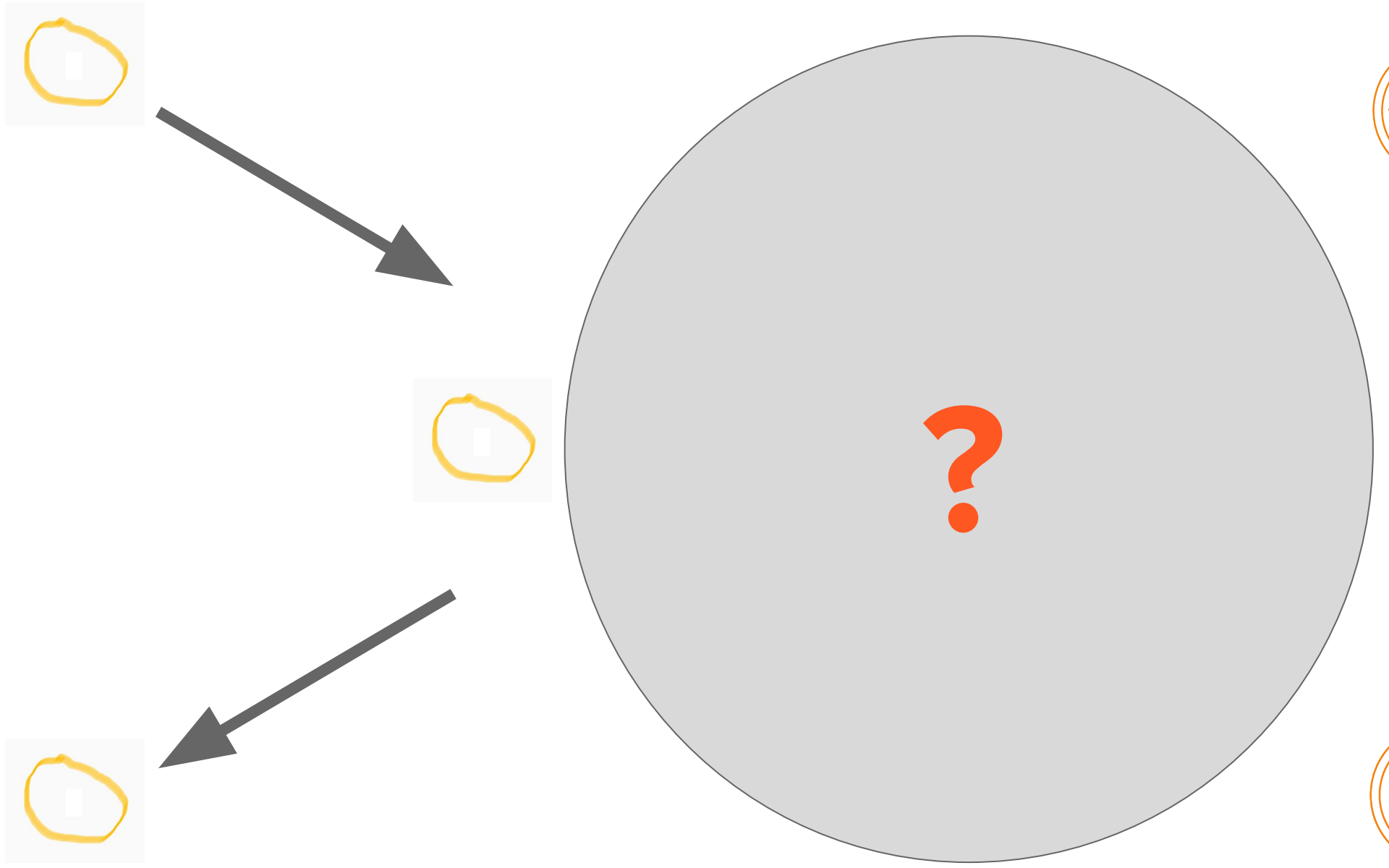
Quantum gravity

- **ħ** – the smaller the particle size, the large its energy
- **G** – too much energy at one place creates a black hole
- **G+ħ** – very energetic particles create black holes







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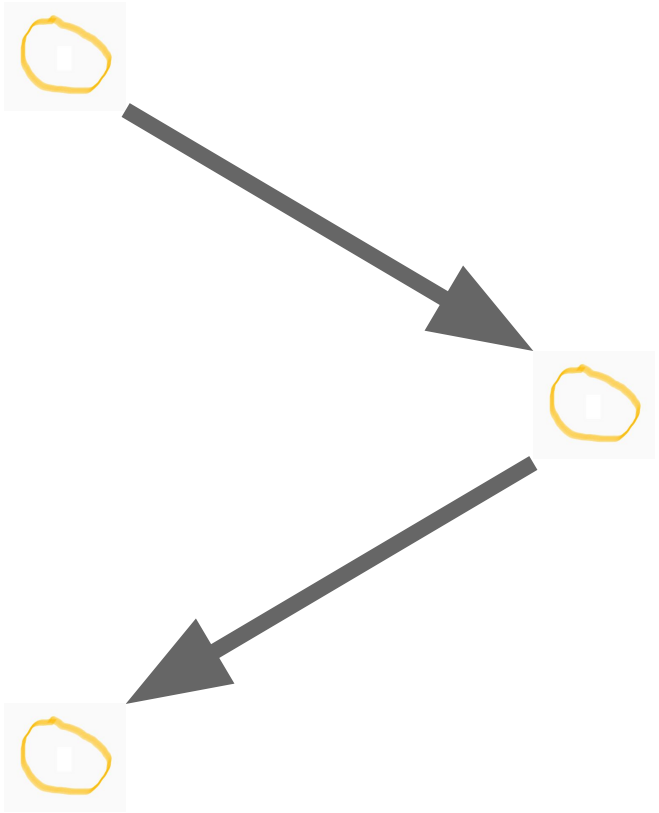
MCMXIX

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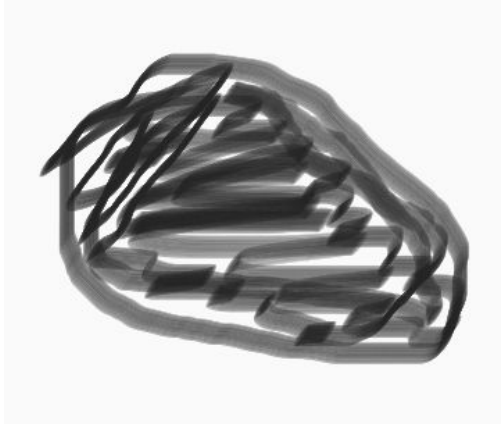
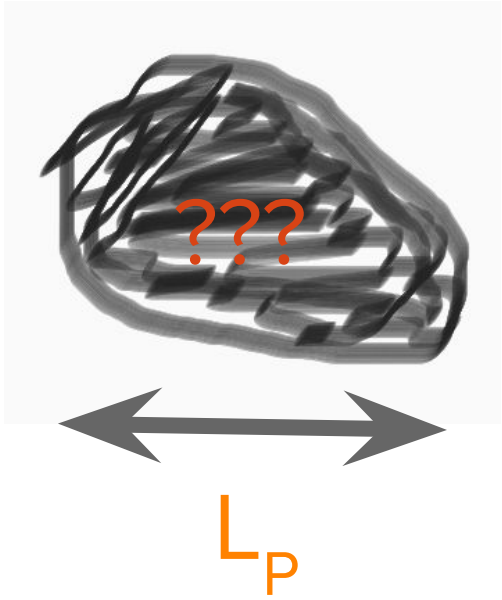



???
L_P

Quantum gravity

- $\hbar$ – the smaller the particle size, the large its energy
- G – too much energy at one place creates a black hole
- G+$\hbar$ – very energetic particles create black holes
- Processes at scales smaller than L_P are hidden from the rest of the world.

Quantum gravity

- ħ - vacuum is full of particles that appear and disappear (quantum fluctuations)

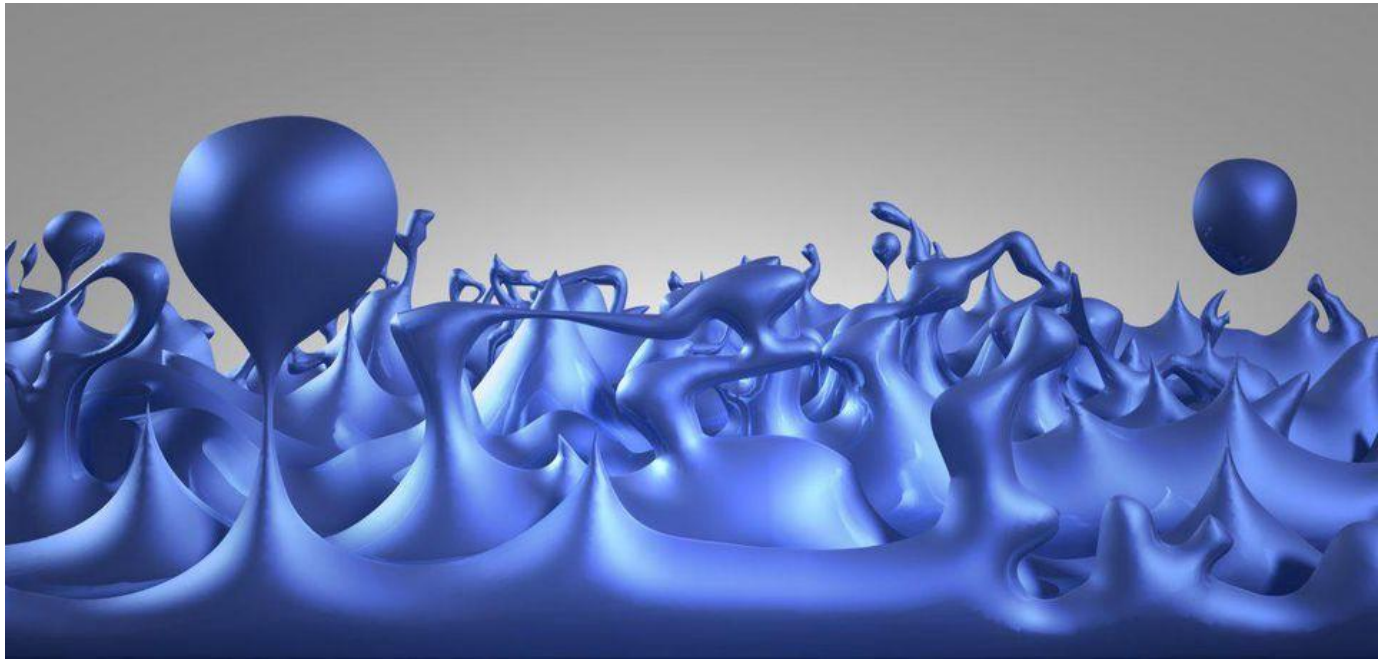


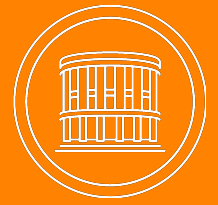
fig.: NASA/CXC/M.WEISS

Quantum gravity

- **ħ** – vacuum is full of particles that appear and disappear (quantum fluctuations)
- **G** – too much energy at one place creates a black hole
- **G+ħ** – vacuum is unstable and disintegrates into a slew of black holes

Problem: very localized energy creates instabilities.

Experiment

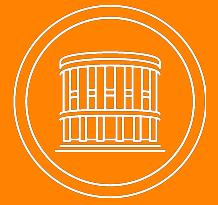


Problem: very localized energy creates instabilities. Which we do not observe.

Solution

- It is not possible to create arbitrarily small object.
- At the distances compared to L_P the space is composed of something.

Quantum structure of spacetime



Quantum structure of spacetime

- Graph paper

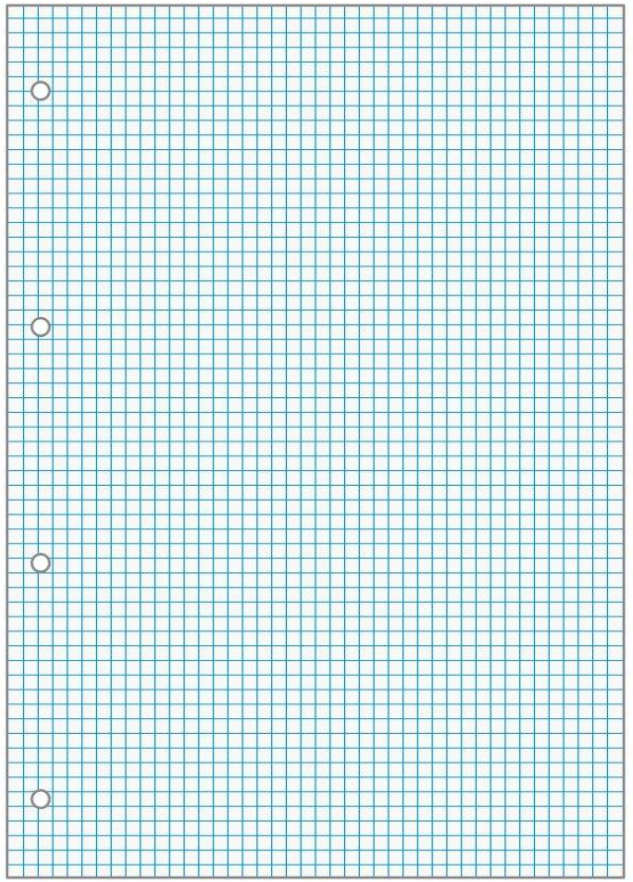


fig.: sevt.sk

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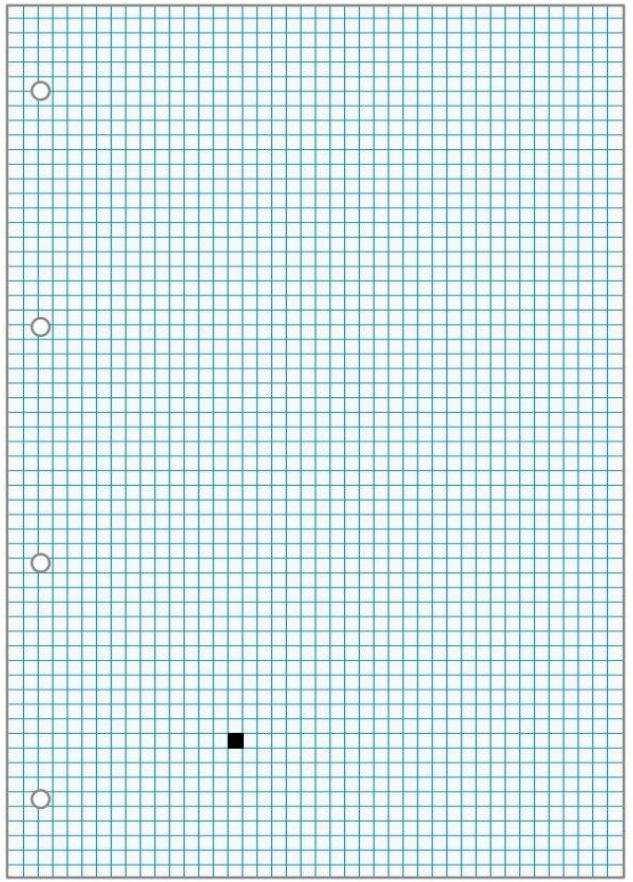
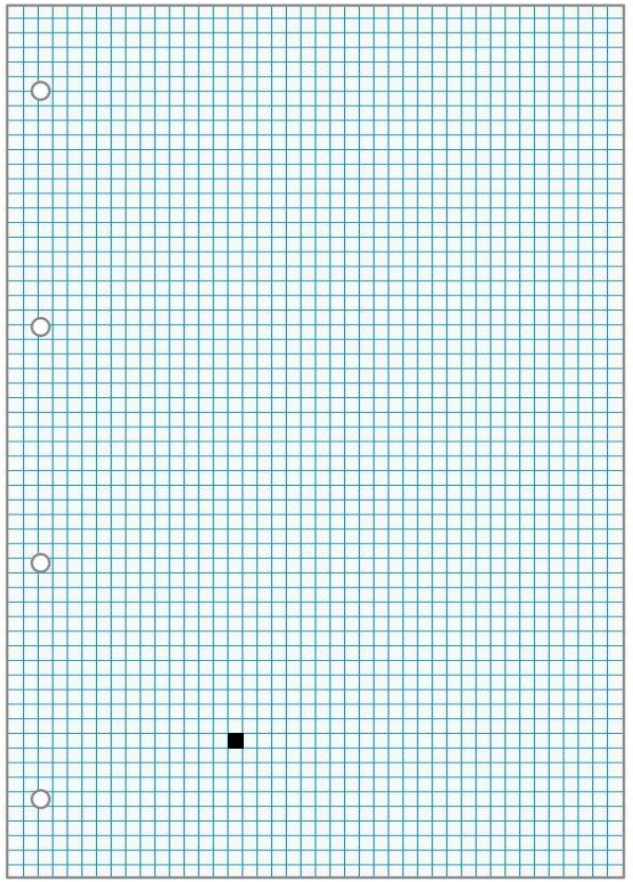


Quantum structure of spacetime

- Graph paper
  - This space has much smaller symmetry.



Quantum structure of spacetime

- Graph paper
- Bubble space

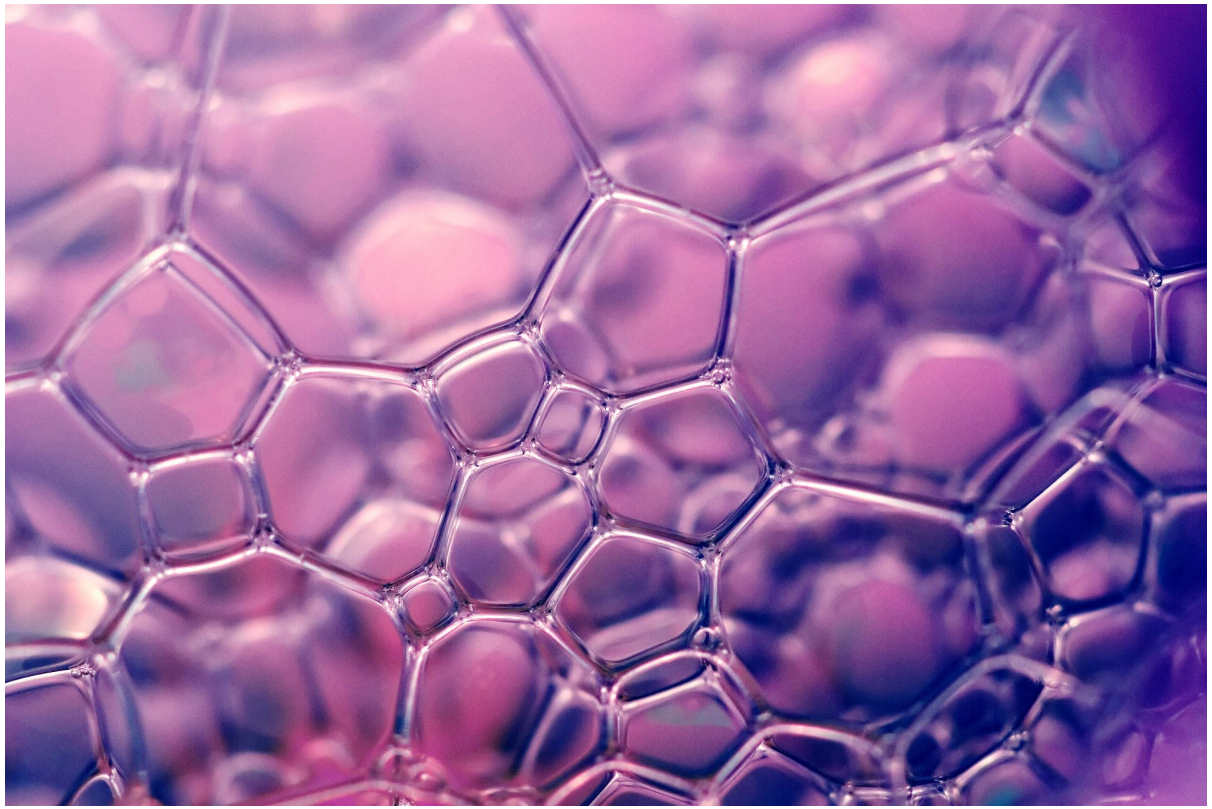


fig.: Umberto Salvagnin, CC BY 2.0

Quantum structure of spacetime

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Quantum structure of spacetime

- Graph paper
- Bubble space
  - This space has continuous symmetries.
  - It is called noncommutative (fuzzy) space.
  - Idea similar to quantum mechanics

$$[x,p]=i\hbar$$

Quantum structure of spacetime

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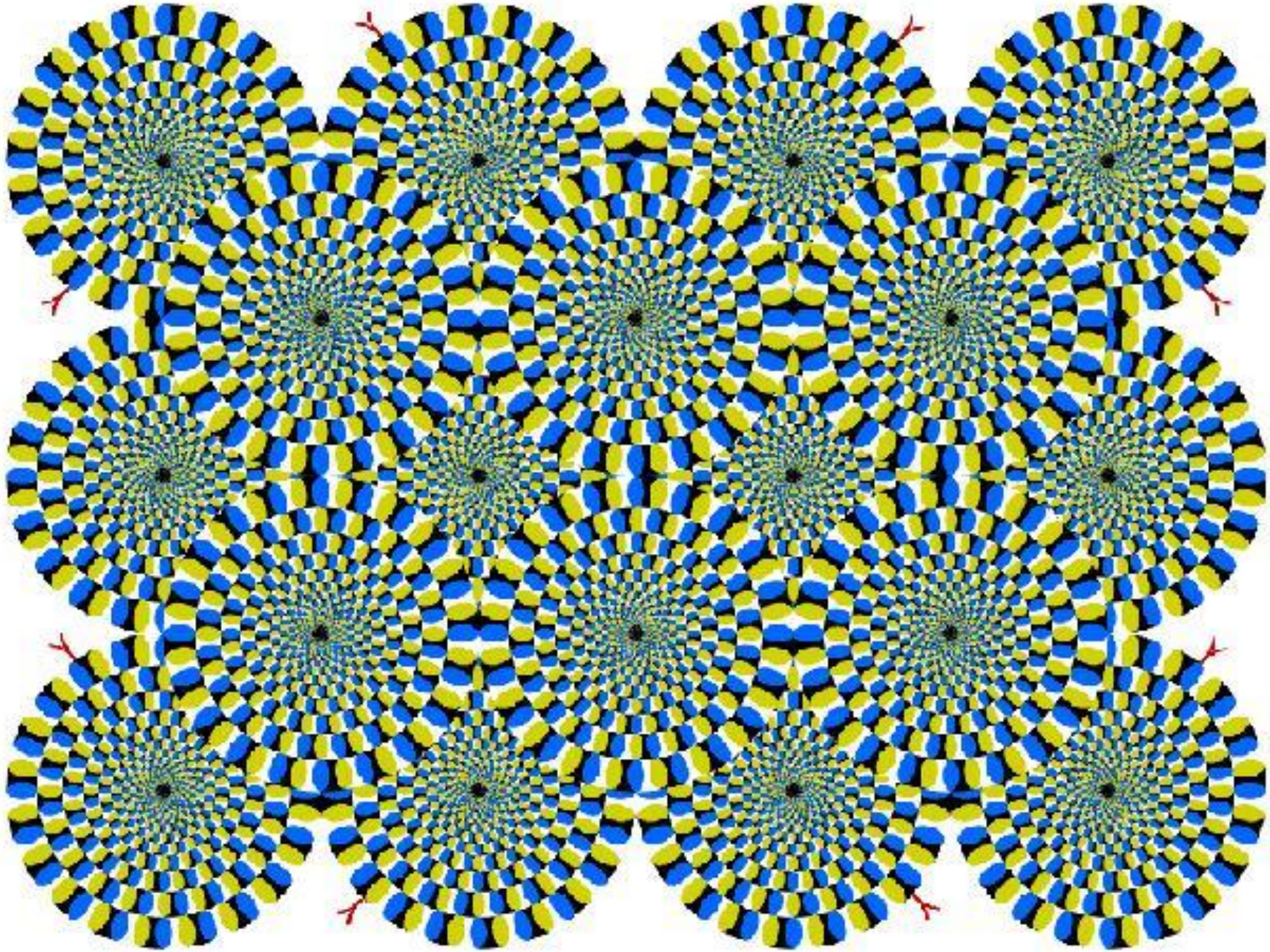
$$[x,y]=i\theta$$



Quantum structure of spacetime

- Graph paper
- Bubble space
- Emergent space
  - Space does not exist.
  - Properties such as location, speed, etc. are a result of interaction with something else.

Can we see the structure of spacetime?

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- **Not** directly. (In the horizont of 100ish years.)

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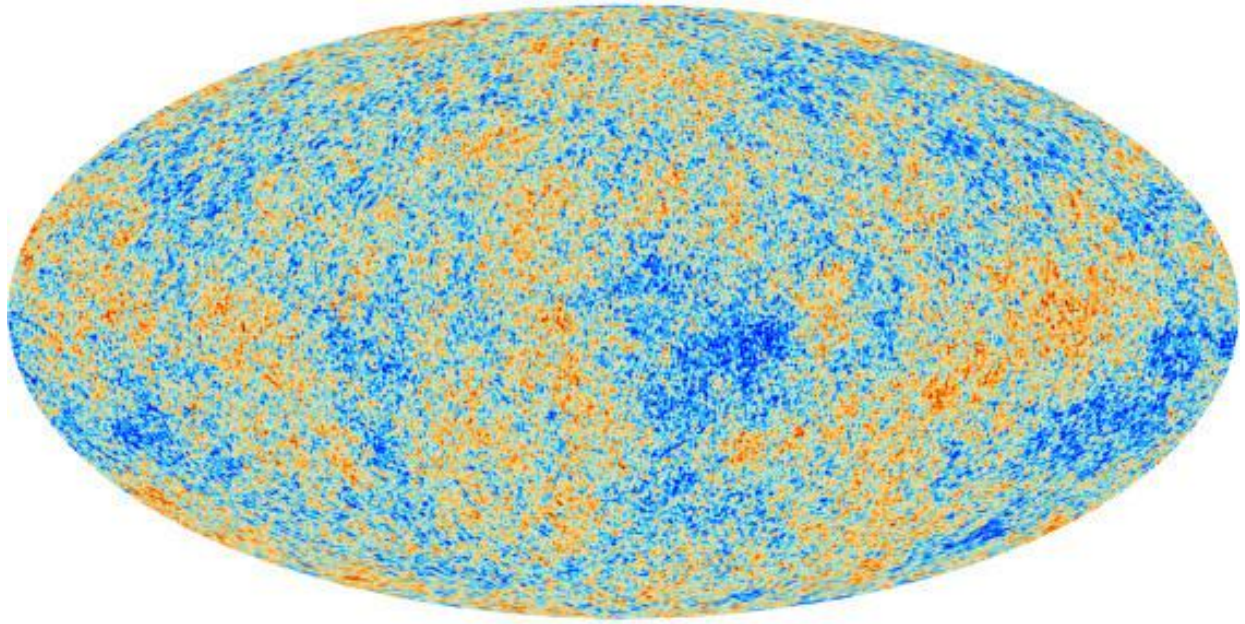

fig.: ESA

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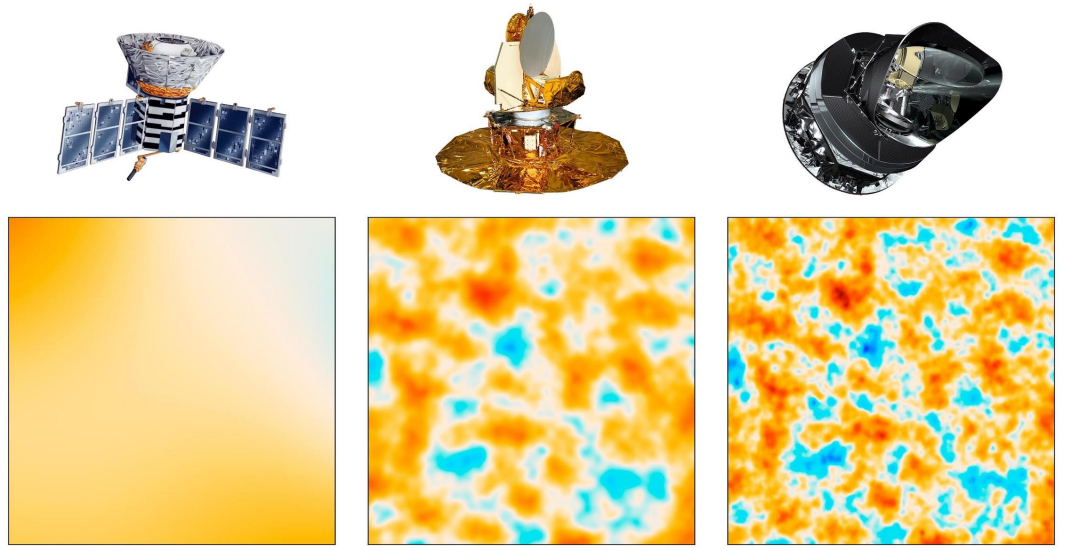

fig.: NASA

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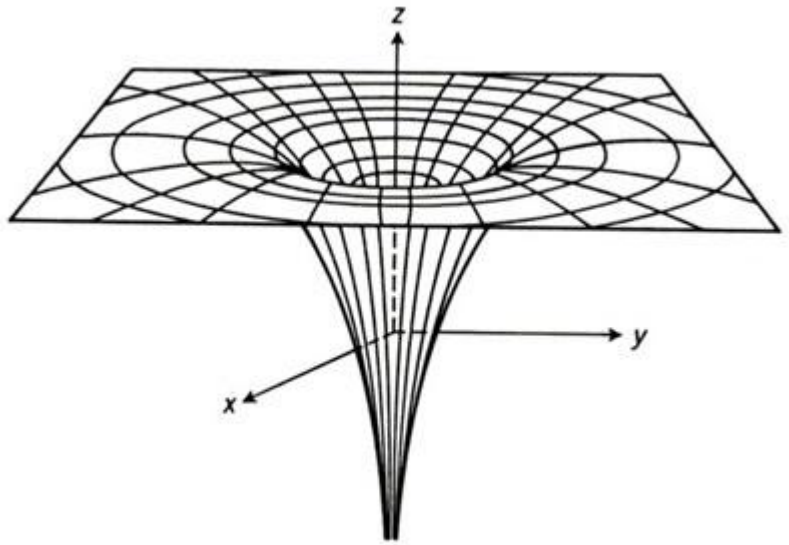

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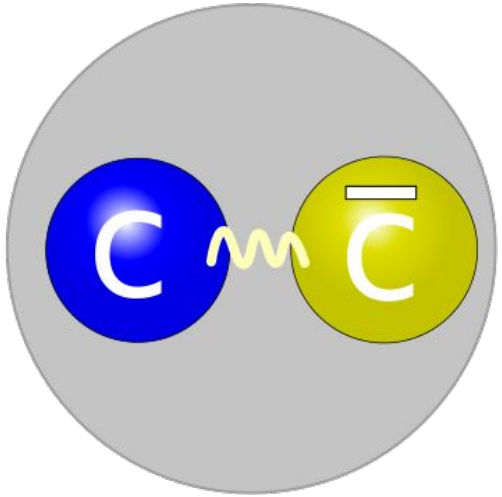

fig.: wiki

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- Requirement of mathematical consistency can have implications at scales much larger than L_P.

Unification of **quantum** mechanics and **gravitation** predicts structure of spacetime.

We are not certain what it is.

Clarifying this will be an important step towards the quantum theory of gravity.

Thank you for your attention!