

# Advanced statistical physics

## Homework 3

first appeared on 11.5.2020, due 30.5.2020 by email to juraj.tekel@gmail.com

*feel free to hand in solutions in slovak*

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**Príklad 1** (Heat capacities quantum correction). What is the change of the heat capacities of the bosonic and fermionic gas due to the quantum correction in the equation of state? What is the corrected Mayer relation?

**Príklad 2** (Multi-critical phase transition.). Let us have a system with the order parameter  $m$  and the free energy given by

$$F(m) = am^2 + gm^{2n}$$

where  $n$  is an integer and  $a$  and  $g$  are some functions of the temperature  $T$ . Determine the phase transition condition and the critical exponents  $\alpha, \beta$

$$m \sim (T_c - T)^\beta, \quad C \sim (T_c - T)^{-\alpha}.$$

**Príklad 3** (Two models of the atmosphere.). Let us have an atmosphere in local thermodynamic equilibrium governed by the collision-less Boltzmann kinetic equation  $Df_0 = 0$ . Under the assumptions of

- isothermic,
- adiabatic,

composition of the gas, what is the dependence of the density of the gas on the height.

**Príklad 4 (Bonus.** Noisy Aristotelian harmonic oscillator.). Let us have a particle which is subject to a random external white noise force and a harmonic oscillator force  $\vec{F} = -k\vec{x}$ . Moreover, particle is in a damping medium with a very large coefficient  $\gamma$ , so we can study the particle in the  $m \rightarrow 0$  limit.

The particle starts its motion from rest at some position away from equilibrium. What is the mean position and the dispersion of the position of the particle?