Diferenciálne rovnice

Vzorcovník k písomnej skúške

Toto sú všeobené vzťahy ktoré si netreba pamätať ale stačí im rozumieť. Ak sa pri riešení niektorého konkrétneho príkladu objaví iný vzorec, ktorý si tiež netreba pamätať ale vedieť odvodiť, bude uvedený v zadní písomky.

$$F(x_1,\dots,x_n,z,p_1,\dots,p_n)=0$$

$$\Downarrow$$

$$\dot{x}_i = \partial_{p_i} F \tag{1a}$$

$$\dot{p}_i = -\partial_{x_i} F - p_i \partial_z F \tag{1b}$$

$$\dot{z} = p_i \partial_{p_i} F \tag{1c}$$

$$F(\vec{x}_0, z_0, \vec{p}_0) = 0 \tag{2a}$$

$$\vec{x}_0 \in \gamma\ ,\ z_0 = f(\vec{x}_0) \tag{2b}$$

$$\frac{\partial z_0}{\partial \tau_j} = p_{i0} \frac{\partial x_i}{\partial \tau_j}\bigg|_{\vec{x}=\vec{x}_0} \tag{2c}$$

$$G_1 = \frac{1}{2}\theta(ct-|x|) \tag{3a}$$

$$G_2 = \frac{1}{2\pi}\frac{\theta(ct-r)}{\sqrt{c^2t^2-r^2}} \tag{3b}$$

$$G_3 = \frac{1}{4\pi|\vec{x}|}\delta(ct-|\vec{x}|) \tag{3c}$$

$$u(x,t) = \frac{1}{2}\left(\phi(x-ct)+\phi(x+ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} dy\psi(y) + \frac{1}{2c}\int_0^t d\tau \int_{x-c(t-\tau)}^{x+c(t-\tau)} dy\, f(y,\tau) \tag{4}$$

$$u(\vec{x},t) = \frac{1}{2\pi c}\frac{\partial}{\partial t}\left(\int_{B(x,ct)} d^2y \frac{\phi(y)}{\sqrt{c^2t^2-|x-y|^2}}\right) + \frac{1}{2\pi c}\int_{B(x,ct)} d^2y\frac{\psi(y)}{\sqrt{c^2t^2-|x-y|^2}} + \\ + \frac{1}{2\pi c}\int_0^t d\tau \int_{B(x,c(t-\tau))} d^2y \frac{f(y,\tau)}{\sqrt{c^2(t-\tau)^2-|x-y|^2}} \tag{5}$$

$$u(\vec{x},t) = \frac{1}{4\pi c^2}\frac{\partial}{\partial t}\left(\frac{1}{t}\oint_{\partial B(\vec{x},ct)} d^2y\,\phi(\vec{y})\right) + \frac{1}{4\pi c^2 t}\oint_{\partial B(\vec{x},t)} d^2y\,\psi(\vec{y}) + \\ + \frac{1}{4\pi c^2}\int_{B(\vec{x},ct)} d^3y \frac{f(\vec{y}, t-|\vec{x}-\vec{y}|/c)}{|\vec{x}-\vec{y}|} \tag{6}$$

$$u(x,t) = \int dy \frac{1}{\left(\sqrt{4\pi t}\right)^n} e^{-\frac{|x-y|^2}{4t}}\phi(y) + \int_0^t d\tau \int dy \frac{1}{\left(\sqrt{4\pi(t-\tau)}\right)^n} e^{-\frac{|x-y|^2}{4(t-\tau)}} f(y,\tau) \tag{7}$$

$$K = \frac{1}{(2-n)\omega_n}\frac{1}{|x|^{n-2}}\ ,\ K = \frac{1}{2\pi}\log|x| \tag{8}$$

$$\frac{\partial K(x-y)}{\partial y_i} = -\frac{1}{\omega_n}\frac{x_i-y_i}{|x-y|^n} \tag{9}$$

$$G(x,y) = K(x-y) - \left(\frac{R}{|y|}\right)^{n-2} K(x-y^*) \tag{10}$$

$$\frac{\partial G}{\partial n_y} = \frac{1}{\omega_n R}\frac{R^2-|x|^2}{|x-y|^n} \tag{11}$$

$$u(r,\theta) = \left(A_0 + B_0 \log\frac{1}{r}\right) + \sum_{k=1}^{\infty}\left(A_k r^k + B_k \frac{1}{r^k}\right)\left(C_k\cos k\theta + D_k \sin k\theta\right) \tag{12}$$

$$u(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[A_{lm}r^l + B_{lm}\frac{1}{r^{l+1}}\right]\left[C_{lm}P_l^m(z) + D_{lm}Q_l^m(z)\right]\left[E_{lm}e^{im\varphi} + F_{lm}e^{-im\varphi}\right] \tag{13}$$

$$u(r,\theta) = \sum_{m=0}^{\infty}\left(A_m J_m\left(\sqrt{\lambda}r\right) + B_m Y_m\left(\sqrt{\lambda}r\right)\right)\left(C_m e^{im\theta} + F_m e^{-im\theta}\right) \tag{14}$$

$$u(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{\sqrt{r}}\left[A_{lm}J_{l+\frac{1}{2}}\left(\sqrt{\lambda}r\right) + B_{lm}Y_{l+\frac{1}{2}}\left(\sqrt{\lambda}r\right)\right]\times \\ \times\left[C_{lm}P_l^m(\cos\theta) + D_{lm}Q_l^m(\cos\theta)\right]\left[E_{lm}e^{im\varphi} + F_{lm}e^{-im\varphi}\right] \tag{15}$$