METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto19 – Príklady 1

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Príklad 1

We have D ducks and H hunters. Probability that a particular duck will not be hit is equal to probability that all hunters are aiming at other ducks. The probability that a hunter is aiming at some specific duck is $1/D$ and thus the probability of not being hit for a duck is

$$P_{\text{not hit}} = \left(1 - \frac{1}{D}\right)^H$$

and the average number of ducks to survive the ordeal is

$$N_{escape} = DP_{\text{not hit}} = D\left(1 - \frac{1}{D}\right)^H = \boxed{\frac{(D-1)^H}{D^{H-1}}} \approx 3.49 \qquad \text{for } D = H = 10$$

Príklad 2

Stepping on the platform will lower it a distance D. This means that the spring constant of the platform spring is given by

$$kD = m_p g.$$

If the lump of clay is removed, then the equilibrium position of the platform would rise a distance A given by

$$kA = m_c g.$$

This would also be the amplitude of the oscillations after the clay is released, so

$$A_i = \frac{m_c g}{k}$$

The frequency of oscillation of the plate without the clay is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_p}}$$

The time for a complete oscillation is

$$T = 2\pi\sqrt{\frac{m_p}{k}}$$

If the clay falls a distance h, then

$$h = \frac{1}{2}gT^2 = 2\pi^2 g\frac{m_p}{k} = 2\pi^2 H.$$

When the plate is at the stating point it is at rest. The clay will hit it with a speed given by

$$v_0 = gT.$$

Conservation of momentum in an inelastic collision will then result in a final speed of the clay + platform system of

$$v_f = v_0\frac{m_c}{m_c + m_p}.$$

The kinetic energy just after collision will be

$$K = \frac{1}{2}(m_c + m_p){v_f}^2.$$

So the amplitude of the resulting oscillations will be given by

$$\frac{1}{2}k{A_f}^2 = \frac{1}{2}(m_c + m_p){v_f}^2$$

or

$$A_f = v_f\sqrt{\frac{m_c + m_p}{k}}$$

Gluing stuff together

$$
\begin{aligned}
\frac{A_f}{A_i} &= \frac{v_f}{A_i}\sqrt{\frac{m_c+m_p}{k}}, \\
&= \frac{v_0}{A_i}\frac{m_c}{m_c+m_p}\cdot\sqrt{\frac{m_c+m_p}{k}}, \\
&= \frac{gTk}{m_c g}\frac{m_c}{m_c+m_p}\cdot\sqrt{\frac{m_c+m_p}{k}}, \\
&= T\sqrt{\frac{k}{m_p+m_c}}
\end{aligned}
$$

And then, combining with our previous expression for T,

$$
\begin{aligned}
\frac{A_f}{A_i} &= 2\pi\sqrt{\frac{m_p}{k}}\sqrt{\frac{k}{m_p+m_c}}, \\
&= 2\pi\sqrt{\frac{m_p}{m_p+m_c}}
\end{aligned}
$$

If, instead, the clay manages to hit the platform at the top of an oscillation, then the distance the clay would fall would only be

$$h - 2A$$

and the time required would be

$$\frac{T}{2} = \pi\sqrt{\frac{m_p}{k}}.$$

Then

$$h - 2A = \frac{1}{2}g\left(\frac{T}{2}\right)^2,$$

where that complicated looking thing is actually

$$\frac{1}{4}h.$$

So

$$
\begin{aligned}
h &= \frac{1}{4}h + 2A, \\
\frac{3}{8}h &= A, \\
\frac{3}{4}\pi^2 D &= A.
\end{aligned}
$$

But from the very first two equations,

$$\frac{m_c}{m_p} = \frac{A}{D} = \frac{3}{4}\pi^2.$$

In this scenario the clay passes the platform three times: once at the highest point, once some distance further on, and once when the at the lowest point. That means that if A were smaller, it might be possible to find a value of A such that the clay just barely touches the platform once before hitting the platform at the bottom. Consequently, this is an *over* estimate for A, and an overestimate for m_c, and therefore, and overestimate for the ratio. A larger ratio would guarantee collision, the actual critical ratio would be smaller than this.

Príklad 3

C… Conservation of mechanical energy can be employed here. This gives $\Delta KE + \Delta PE = 0$ and we note that we have two forms of kinetic energy (translational and rotational). Incorporating both, we write $\Delta KE_{tr} + \Delta KE_{rot} + \Delta PE = 0 \rightarrow \left(\tfrac{1}{2} m v_f^2 - 0\right) + \left(\tfrac{1}{2} I\omega_f^2 - 0\right) + \left(0 - \tfrac{1}{2} k x_i^2\right) = 0$. Noting from the equation sheet that $I_{cyl} = \frac{1}{2} MR^2$, we can write, using $v = r\omega$, that

$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2 = \tfrac{1}{2} k x_i^2 \rightarrow m v^2 + \tfrac{1}{2} m v^2 = k x_i^2 \Rightarrow v_{cm} = \sqrt{\tfrac{4}{3}\tfrac{k}{m}}\; x_i = 1.15 \,\tfrac{m}{s}$$

Príklad 4

46. Podobne ako v úlohe 45 možno nájsť "zrkadlový" náboj q' (z vonkajšej strany guľovej plochy), ktorého veľkosť je $q' = -(a/\delta)q$ a je umiestnený vo vzdialenosti $d = a^2/\delta$ od stredu guľovej plochy. Náboj q bude priťahovaný ku guľovej ploche silou

$$f = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{(d-\delta)^2} = \frac{q^2}{4\pi\varepsilon_0}\frac{a\delta}{\left(a^2-\delta^2\right)^2}$$

ktorá nezávisí od elektrického stavu gule, t. j. či je guľa uzemnená, izolovaná, nabitá alebo nenabitá.

Príklad 5

(a) Let the length of string on the table be r and the length hanging below be y. The mass m_1 is described by the polar coordinates (r, ϕ). The fixed length constraint is $y + r = \ell$. The Lagrangian is

$$\begin{aligned} L &= \tfrac{1}{2} m_1\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \tfrac{1}{2} m_2 \dot{y}^2 + m_2 g y \\ &= \tfrac{1}{2}(m_1 + m_2)\dot{r}^2 + \tfrac{1}{2} m_1 r^2 \dot{\phi}^2 + m_2 g(\ell - r)\ . \end{aligned}$$

(b) The angular momentum is conserved: $\dot{p}_\phi = 0$, with

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m_1 r^2 \dot{\phi}\ .$$

The equation of motion for r yields

$$\begin{aligned} (m_1 + m_2)\ddot{r} &= m_1 r \dot{\phi}^2 - m_2 g \\ &= \frac{p_\phi^2}{m_1 r^3} - m_2 g \equiv -\frac{\partial U_{\text{eff}}}{\partial r}\ , \end{aligned}$$

where the effective potential is

$$U_{\text{eff}} = \frac{p_\phi^2}{2 m_1 r^2} + m_2 g r\ .$$

The condition for stationary m_2 is $\dot{r} = \ddot{r} = 0$, which requires $U'_{\rm eff}(r) = 0$. This, in turn, has the solution $r = a$, with

$$a = \left(\frac{p_\phi^2}{m_1 m_2\, g}\right)^{1/3} .$$

(c) The tension in the string along the table is radial, hence there are no torques, and p_ϕ is conserved.

(d) We write $r = a + \eta$ and expand the equation of motion:

$$(m_1 + m_2)\,\ddot{\eta} = -U''_{\rm eff}(a)\,\eta + \mathcal{O}(\eta^2) .$$

The solution is

$$\eta(t) = \eta_0 \cos\big(\omega t + \delta\big) ,$$

where the oscillation frequency is

$$\omega = \sqrt{\frac{U''_{\rm eff}(a)}{m_1 + m_2}} = \sqrt{\frac{3m_2\, g}{(m_1 + m_2)a}} .$$