METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto19 – Príklady 3

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Príklad 1

23 Pozrime sa na to, aká sila pôsobí na podložku. Musí to byť súčet reakcie na tiažovú silu misky $F_{\text{miska}} = Mg$ a tiažovú silu vody $F_{\text{voda}} = \frac{2}{3}\pi R^3 \rho g$.

Teraz sa zamyslime, aká sila spôsobí zdvihnutie misky. Treba si uvedomiť, akú úlohu zohráva tlak vody v dolnej podstave. Podľa tretieho Newtonovho zákona musí rovnaká tlaková sila, aká pôsobí na podložku, pôsobiť aj od podložky na kvapalinu. Práve tento tlak s veľkosťou $p = h\rho g$ spôsobuje zdvihnutie misky. Pre silu potom platí $F = pS = \rho g \pi R^3$. Teraz už len stačí dať sily do rovnosti a vyjadriť výslednú hmotnosť misky ako

$$\rho g \pi R^3 = Mg + \frac{2}{3}\pi R^3 \rho g,$$

odkiaľ

$$M = \frac{1}{3}\pi R^3 \rho.$$

Príklad 2

One can express the net radiative transfer, I_{net}, in terms of the left and right portions of the system.

$$I_R = \epsilon_L \sigma T_L^4 + (1 - \epsilon_L) I_L$$

$$I_L = \epsilon_R \sigma T_R^4 + (1 - \epsilon_R) I_R$$

$$I_{\text{net}} = I_R - I_L$$

By looking at the quantity $\epsilon_R I_R - \epsilon_L I_L$ and isolating $I_R - I_L$, one finds...

$$\boxed{I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{\epsilon_R + \epsilon_L - \epsilon_L \epsilon_R}}$$

For the infinite series approach, consider the contribution from each sequence. That is.

$$I'_R = \epsilon_R \epsilon_L \sigma T_L^4 (1 + (1 - \epsilon_R)(1 - \epsilon_L) + ((1 - \epsilon_R)(1 - \epsilon_L))^2 + ...)$$

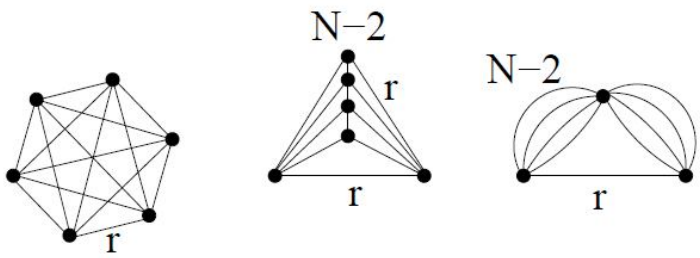
and the same for I'_L. The infinite series term is just the geometric series with a value $(1 - (1 - \epsilon_R)(1 - \epsilon_L))^{-1}$. Therefore, the net effect is

$$I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{1 - (1 - \epsilon_R)(1 - \epsilon_L)}$$

Which is the same as above.

Príklad 3

If the number of point is N, then we have two points (IN and OUT) that are connected directly. Then we also have $N-2$ points that are connected to the IN and OUT points and all interconnected among themselves. Since all resistors are the same, all the $N-2$ points have same potential, and can be connected into one point. In other words, there is no current flowing between these middle points due to symmetry.



So our equivalent electric configuration is ($N-2$ parallel resistors in series with $N-2$ parallel resistors) in parallel with one resistor.

If the number of points is N the resistance between any two points is then

$$\boxed{R = \left(\frac{1}{r} + \frac{N-2}{2r}\right)^{-1} = \frac{2}{N}r}$$

Príklad 4

#12: GRADUATE MECHANICS

PROBLEM: Starting from rest at $(x, y) = (0, 0)$, a particle slides down a frictionless hill whose shape is given by the equation $y = -ax^n$, $a > 0$ and $n > 0$. Determine the range of allowed n for which the particle leaves the surface, and the x location at which this occurs. Assume gravity is constant, in the $-y$ direction.

SOLUTION:

$$E = 0 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy$$

$$y = -ax^n \;\Rightarrow\; \dot{y} = -nax^{n-1}\dot{x}$$

$$\Rightarrow\; \dot{x}^2 = \frac{2gax^n}{1+n^2a^2x^{2(n-1)}}$$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$$2\not{\dot{x}}\,\ddot{x} = \frac{\partial}{\partial t}\left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}}\right) = \not{\dot{x}}\,\frac{\partial}{\partial x}\left(\frac{2gax^n}{1+n^2a^2x^{(2n-2)}}\right) = 0$$

$$\Rightarrow\; \frac{nx^{n-1}}{1+n^2a^2x^{2(n-1)}} - \frac{2(n-1)n^2a^2x^{3n-3}}{(1+n^2a^2x^{2n-2})^2} = 0$$

$$nx^{n-1} + n^3a^2x^{3n-3} - 2(n-1)n^2a^2x^{3n-3} = 0$$

$$\Rightarrow\; a^2x^{2n-2} = \frac{1}{n(n-2)}.$$ Real finite solution only for $n > 2$.

Príklad 5

SOLUTION:

Ohm's law for a wire of length l gives the voltage $V = I\rho l/A$. Therefore, the electric field strength in the wire is $E = V/l = I\rho/A$. Due to the difference in the resistivity of the materials the electric field strength has to be different in material 1 and material 2. According to Gauss's law, the difference in the electric field strengths implies an accumulation of charge at the boundary of the two materials. The net accumulated charge is

$$Q = \varepsilon_0 A(E_2 - E_1) = \varepsilon_0 I(\rho_2 - \rho_1), \tag{5}$$

where ε_0 is the electric constant.