METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto19 – Príklady 2

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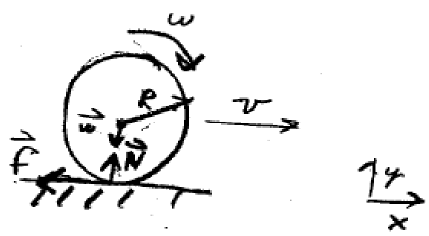
Príklad 1

#1

a) Torque caused by Friction leads to rolling

b) $V = \omega R$

c) Eqns of C.O.M.



① $m\ddot{y} = W - N = 0$

$\quad = Mg - N$

$\therefore N = Mg$

② $m\ddot{x} = -f$

$\quad = -\mu N$

$\quad = -\mu Mg$

$\therefore V = \dot{x} = \int_0^t dt' \, \ddot{x} = v_0 - \mu g t$

Eqn about C.O.M

$I\ddot{\theta} = Rf$

or $\frac{2}{5} MR^2 \ddot{\theta} = R\mu Mg$

$\ddot{\theta} = 5/2 \, \frac{\mu g}{R}$

$\omega = \dot{\theta} = \int_0^t dt' \, \ddot{\theta} = 5/2 \, \frac{\mu g}{R} t$

Rolling w/o slipping when $V = \omega R$, or

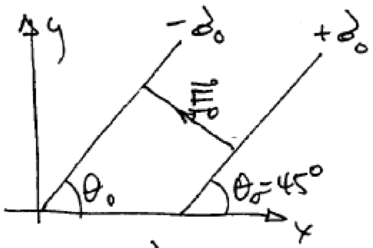
$v_0 - \mu g T = 5/2 \, \mu g T$

or $T = 2/7 \, v_0/\mu g$

Príklad 2

PART I : PROBLEM #3

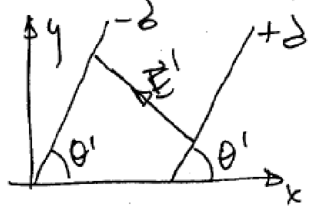
a)



In S_0, $|\vec{E}_0| = \frac{\sigma_0}{\epsilon_0}$, where

$$E_{0x} = -\frac{\sigma}{\sqrt{2}\,\epsilon_0}, \quad E_{0y} = +\frac{\sigma}{\sqrt{2}\,\epsilon_0}$$

b)



In S, $E_x' = E_{0x} = -\frac{\sigma_0}{\sqrt{2}\,\epsilon_0}$ (∥ component preserved)

but $E_y' = \gamma E_{0y} = +\frac{\gamma \sigma_0}{\sqrt{2}\,\epsilon_0}$ (⊥ component to relative motion)

and $E' = \sqrt{\frac{1+\gamma^2}{2}}\,\frac{\sigma_0}{\epsilon_0}$.

c) $dy' = dy_0$ ⊥ to relative motion

$dx' = dx_0/\gamma$ length contracted along motion.

$\therefore \tan\theta' = \frac{dy'}{dx'} = \gamma \frac{dy_0}{dx_0}$, but $\frac{dy_0}{dx_0} = \tan 45^\circ = 1$

$\therefore \theta' = \tan^{-1}\gamma$ is angle of plates wrt x-axis

d) $$\frac{d\vec{r}' \cdot \vec{E}'}{r' E'} = \frac{\left(\frac{1}{\gamma}, 1\right)(-1, \gamma)}{\sqrt{1+\frac{1}{\gamma^2}}\sqrt{1+\gamma^2}} = \frac{\gamma^2 - 1}{\gamma^2 + 1} \neq 0$$

$\therefore \vec{E}'$ is <u>not</u> perpendicular to plates in S.

(a) Write down the partition function for this system.

$$Z = \frac{1}{N!}\left(\frac{2\pi m k T}{h^2}\right)^{\frac{3N}{2}} A^N \left[\int_{z_0}^{\infty} e^{-\beta m g z}\, dz\right]^N$$

$$= \frac{1}{N!}\left(\frac{2\pi m k T}{h^2}\right)^{\frac{3N}{2}} \left(\frac{A e^{-\beta m g z_0}}{\beta m g}\right)^N$$

(b) Use this partition function to calculate the mean energy and the heat capacity, C_v, of the gas.

$$\underline{\bar{E}} = -\frac{\partial \ln Z}{\partial \beta} = \frac{3}{2}\frac{N}{\beta} + \frac{N}{\beta} + m g z_0$$

$$= \underline{\frac{5}{2} N k_B T + N m g z_0}$$

$$\underline{C = \frac{5}{2} N k_B}$$

(c) Use the partition function to compute the force on the bottom of the container.

$$\underline{X_{bott}} = \frac{1}{\beta}\frac{\partial \ln Z}{\partial z_0} = \underline{-Nmg}$$

-- just the weight of the gas

Príklad 4

Solution to Stern-Gerlach Problem After the first apparatus, the state of the system is $\chi = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Lets define the beam direction to be along the y axis. There are many ways to solve this problem. One way is to find the eigenstates of S_u then dot them into χ.

$$S_u = \cos\theta S_z + \sin\theta S_x = \hbar\left[\cos\theta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}\right] = \hbar \begin{pmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{2}} & 0 \\ \frac{\sin\theta}{\sqrt{2}} & 0 & \frac{\sin\theta}{\sqrt{2}} \\ 0 & \frac{\sin\theta}{\sqrt{2}} & -\cos\theta \end{pmatrix}$$

We know the eigenvalues of S_u will be 0, $\pm\hbar$ so we have the eigenvalue equation.

$$\begin{pmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{2}} & 0 \\ \frac{\sin\theta}{\sqrt{2}} & 0 & \frac{\sin\theta}{\sqrt{2}} \\ 0 & \frac{\sin\theta}{\sqrt{2}} & -\cos\theta \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

This is straightforward but tedious.

We may use a rotation operator to solve the problem in one step. The rotation operator is

$$R_y(\theta) = e^{i\theta L_y/\hbar} = e^{i\theta L_y/\hbar} = \sum_{n=0}^{\infty} \frac{\left(\frac{i\theta L_y}{\hbar}\right)^n}{n!}$$

$$e^{i\theta L_y/\hbar} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin(\theta)\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} + \frac{1}{2}(\cos(\theta) - 1) \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ -\frac{1}{\sqrt{2}}\sin\theta & \cos\theta & \frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}$$

Using the u-axis to define the basis states we then have

$$\chi' = R_y(\theta)\chi = \frac{1}{2}\begin{pmatrix} 1-\cos\theta \\ \sqrt{2}\sin\theta \\ 1+\cos\theta \end{pmatrix}$$

The sum of the squares is 1 as it should be. So, the relative intensities are the squares of the amplitudes.

$$Intensity \propto \frac{1}{4}\begin{pmatrix} (1-\cos\theta)^2 \\ 2\sin^2\theta \\ (1+\cos\theta)^2 \end{pmatrix}$$

Its easier to rotate about the z-axis since the rotation matrix is just.

$$R_z(\theta) = \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}$$

If we redefine the beam direction to be the z direction, say the intitial apparatus separation is along x, then we just need to find the spin down state along the x axis.

$$\chi = \begin{pmatrix} \frac{-1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{2} \end{pmatrix}$$

$$\chi' = \begin{pmatrix} \frac{-1}{2}e^{i\theta} \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{2}e^{-i\theta} \end{pmatrix}$$

Now dot this back into three S_x eigenstates.

$$\psi_+ = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix} \qquad \psi_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad \psi_- = \begin{pmatrix} \frac{-1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{2} \end{pmatrix}$$

$$I_+ = \left|\frac{1}{2} - \frac{1}{2}\cos\theta\right|^2$$
$$I_0 = \left|\frac{i\sin\theta}{\sqrt{2}}\right|^2$$
$$I_- = \left|\frac{1}{2} + \frac{1}{2}\cos\theta\right|^2$$

Same answer.