Cvičenie 14.3.2019

Príklad 1

Príklad 1. A sphere with radius r and moment of inertia $I = \frac{2}{5}mr^2$ compresses a spring with spring constant k a distance x.

    a. How far must the spring be compressed so that the sphere rolls around the interior of a circular loop of radius R, depicted in the figure above, while always remaining in contact with the track?

   b. In this case described, what height will the sphere reach at the end of the track?
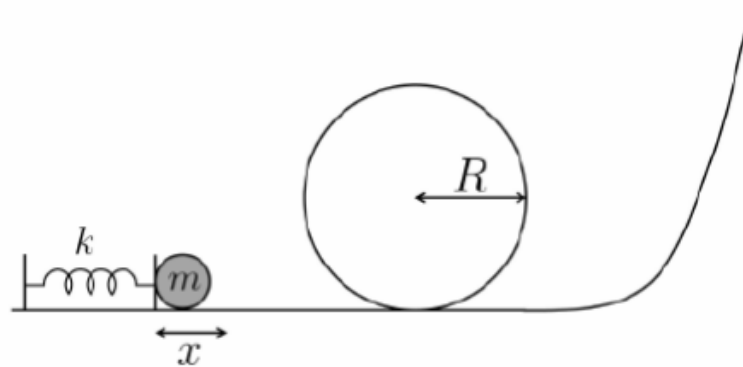
Príklad 2

Príklad 2.   a. A wok, the famous Asian round-bottomed cooking vessel, can be modelled as a uniform hollow semi-sphere. Initially, you have a wok on your table with the open side down. What is the minimum energy needed to flip the wok such that in the end it has the open side up? Assume that the wok is made of a metal sheet with surface density σ and that the gravitational acceleration is g.

    b. If we colonize other planets we will want to move easily between planets and their moons. Assume that you have a planet of mass M_P and at a distance r there is a moon of mass M_M, and you want to send a package of mass δ, from the planet to the moon. Assume that both the planet and the moon have their mass perfectly spherically distributed and that the radius of the planet is R_P and that of the moon is R_M. What is the minimum energy required so that you can transport the package between the planet and the moon? Ignore any form of friction, any gravitational effects caused by any other body around the planet, and any corrections due to general relativity.
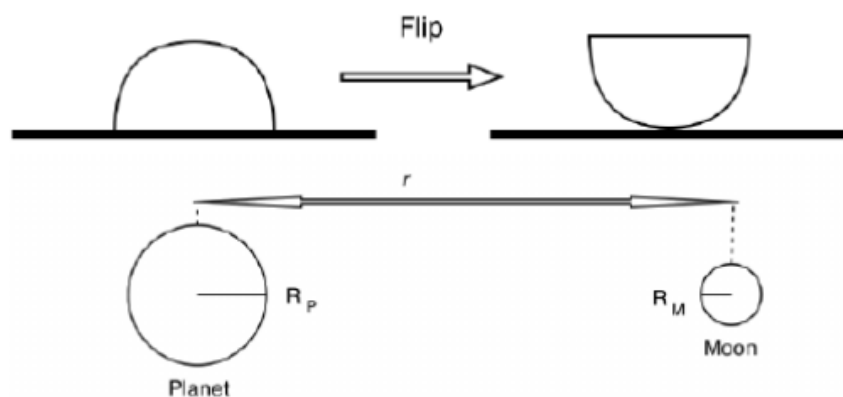
Príklad 3

Uncharged metallic plate of area S and thickness d is distance r from a point charge q. Its surface is perpendicular to the vector **r**. Find the force of attraction of the plate towards the charge. $d \ll \ell \ll r$ where $\ell \sim \sqrt{S}$.
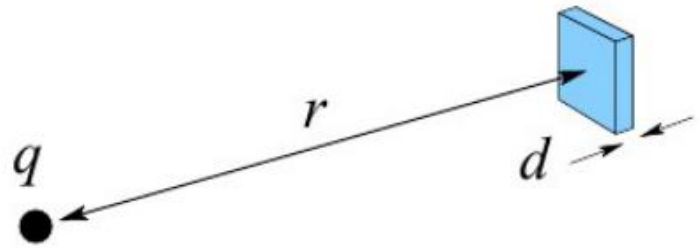
Príklad 4

PROBLEM: A quasi-particle has the following Hamiltonian for motion in two dimensions (x, y):

$$H(x, y, p_x, p_y) = p_x^2 + x^2 p_y^2 + x^2 y^2.$$

Find $x(t)$, $y(t)$ for initial conditions $x(0) = 0$, $p_x(0) = 1$, $y(0) = 0$, $p_y(0) = 1$.
Hint:

$$\int \frac{dy}{\sqrt{1-y^2}} = \sin^{-1} y.$$

Príklad 5

Consider a cylindrical capacitor of length L. The capacitor consists of an inner conducting wire of radius a and an outer conducting shell of radius b. The space between $(a < r < b)$ is filled with a non-conducting material which has a dielectric constant ϵ. In all cases, neglect any end effects or fringing fields.

(a) What is the value of the electric field as a function of the radial position, r, when the capacitor has charge Q on it?

(b) What is the capacitance?

(c) Now suppose the dielectric is pulled out partially while the capacitor is connected to a battery of constant potential V. What is the force required to hold the dielectric in this position?