METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto19 – Príklady 3

Cvičenie 21.3.2019

Príklad 1

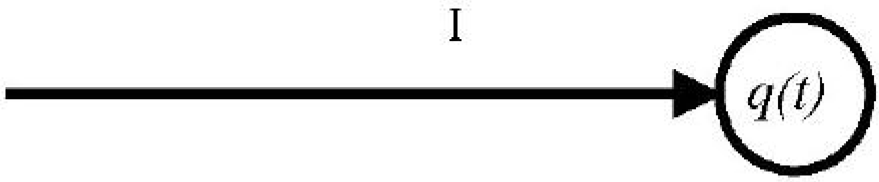
PROBLEM: This problem deals with displacement current.

(a) Consider the displacement current $j_d = \frac{1}{4\pi}\frac{\partial E}{\partial t}$ of an electromagnetic field, and using Maxwell's equations show that the sum $\boldsymbol{J} = \boldsymbol{j} + \boldsymbol{j}_d$ is divergenceless: $\nabla \cdot \boldsymbol{J} = 0$. (Here, $\boldsymbol{j}$ is the current of charges.)

(b) A conducting sphere of radius a is being charged through a straight wire, carrying current I, so that the charge on the sphere q obeys $\dot{q} = I$. Assuming a symmetric distribution of charge over the sphere's surface, find the electric field outside the sphere. Determine also the displacement current, and verify the conservation law $\nabla \cdot \boldsymbol{J} = 0$.

(c) Using the Ampére-Maxwell law in the form $\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J}$, and taking advantage of the cylindrical symmetry in this problem, find the magnetic field everywhere in space.

(d) By appropriately limiting your result from (c), verify that close to the wire, the answer has the familiar form for an infinite straight wire.



Príklad 2

PROBLEM: A hydrogen atom in its ground state is placed in an electric field $\boldsymbol{E}(t) = \boldsymbol{E}_0 \cos(\omega t)$, where $\omega > me^4/2\hbar^3$. Find the probability per unit time that the atom will be ionized. You should assume that the wavefunctions of the electron in the ionized states are plane waves. The ground state wavefunction of hydrogen is $\psi_0(r) = (\pi a_B^3)^{-1/2} \exp(-r/a_B)$, where $a_B = \hbar^2/me^2$ is the Bohr radius.

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7. Two equal containers, each of volume V, contain ideal gases at temperature T, pressure p. In container 1, the gas consists of N_1 molecules of gas α and M_1 of gas β in container 2, N_2 and M_2 respectively - note that $N_1 + M_1 = N_2 + M_2$. Derive an expression for the entropy of mixing, i.e. the entropy gain obtained by allowing the containers to freely mix. Evaluate the two limiting cases (1) $N_1 = N_2$ and (2) $N_1 = M_2$.

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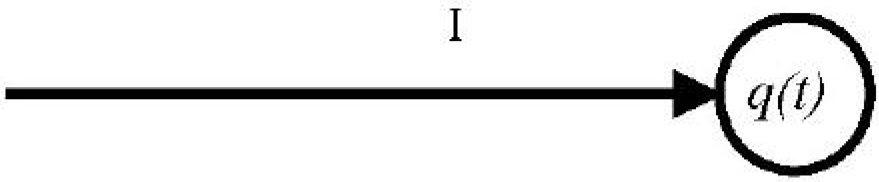

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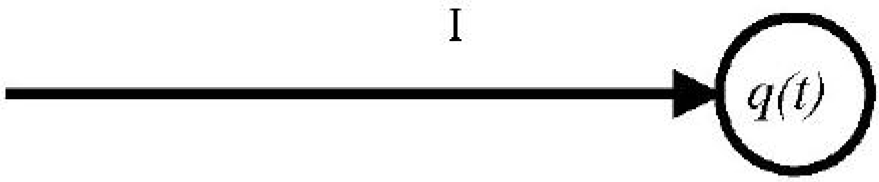

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