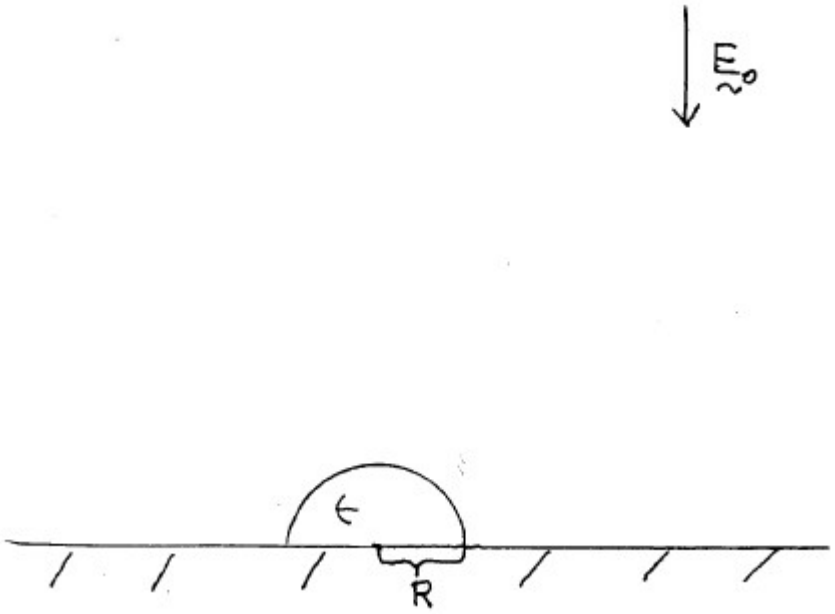
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto19 – Príklady 4

Cvičenie 9.5.2019

Príklad 1

PROBLEM: A solid piece of dielectric is characterized by a permittivity epsilon and has the shape of a hemisphere of radius R. The hemisphere is glued flat side down to a grounded conducting plane (see figure). High above the plane, a uniform electric field E_0 is directed vertically down. Determine the electric potential in and around the piece of dielectric.



Môže sa zísť Riley, Hibson, Bence: Mathematical methods for Physics and Engineering, časť 21.3

Príklad 2

A two level system has as its Hamiltonian

$$\mathcal{H} = \begin{pmatrix} 0 & ig \\ -ig & \Delta \end{pmatrix}$$

in some basis. At time zero, the quantity D, described by

$$D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

is measured and found to have the value zero.

1. What is the probability that a measurement of D at a later time t will yield the value one?

2. If a measuring apparatus monitors the value of D continuously, what is the probability that its value will be one at the later time t?

Príklad 3

PROBLEM: The figure below illustrates a simple model of the surface of a two-dimensional 'solid' confined to a square lattice. The two ends of the surface are N lattice sites apart, with $N \gg 1$. The surface energy is proportional to the surface length, with energy $\varepsilon > 0$ per lattice length. The surface height can change by at most one lattice length at a time. (Overlaps are forbidden, so that outward-pointing surface normals never point downward.) Thus, the surface can be modeled by a Hamiltonian

$$H = \varepsilon \sum_{i=1}^{N} (1 + \sigma_i^2) \ ,$$

where $\sigma_i = +1$, 0, or -1 depending on whether the i^{th} 'column' contains a step up, no step, or a step down for the surface.

(a) Explain why this is Hamiltonian properly reflects the surface energy described above.

(b) Find the partition function $Z(T)$ for the surface.

(c) Find the free energy $F(T)$ for the surface, and sketch its temperature dependence. Physically interpret your result in the limits $k_{\text{B}}T \ll \varepsilon$ and $k_{\text{B}}T \gg \varepsilon$.

(d) Find the total length of the surface as a function of temperature, and sketch its temperature dependence.