METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima18 – Príklady 1

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Cvičenie 4.10.2018

Príklad 1

(2.) na svete je vraj $1,3 \times 10^9\ km^3$ vody

$= 1,3 \times 10^{21}\ l$

v novom pohári $0.5l / 1,3 \times 10^{21} l \approx \frac{1}{3} \times 10^{-21}$

z pôvodného pohára

v pohári je 27 molov vody $(500g / (16\frac{g}{m} + 2 \cdot 1 \frac{g}{m}))$

$1 mol = 6 \times 10^{23}$ častíc $\Rightarrow$ pohár vody $1,62 \times 10^{25}$ častíc

a teda $1,62 \times 10^{25} \times \frac{1}{3} 10^{-21} = \frac{1}{2} 10^4 \sim$ tisíce

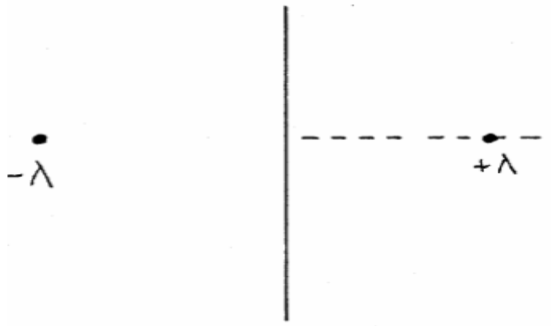
Príklad 2

. **A…** The efficiency of the engine is computed as $e = \left|\frac{W}{Q_h}\right|$. The work done is related to the area the curve which is given as $(2P_0 - P_0)\left(\frac{3}{2}V_0 - V_0\right) = \frac{1}{2}P_0V_0 = \frac{1}{2}nRT_0$. The heat that enters the engine comes during stages $A \to B$ and $B \to C$. To determine the heat for each process, we use $Q_{AB} = nc_v\Delta T$ and $Q_{BC} = nc_p\Delta T$. From the ideal gas equation, the temperature doubles from A to B because the pressure doubles at constant volume. By the same logic, the temperature increases by 3/2 from B to C since the pressure is constant with an increasing volume. We then write $Q_{AB} = nc_v\Delta T = n\left(\frac{3}{2}R\right)(2T_0 - T_0) = \frac{3}{2}nRT_0$. Likewise, $Q_{BC} = nc_p\Delta T = n\left(\frac{5}{2}R\right)(3T_0 - 2T_0) = \frac{5}{2}nRT_0$. This makes the total heat added equal to $Q_h = \frac{5}{2}nRT_0 + \frac{3}{2}nRT_0 = 4nRT_0$. Hence, the efficiency is $e = \left|\frac{W}{Q_h}\right| = \frac{1/_2 nRT_0}{4nRT_0} = \frac{1}{8}$.

Príklad 3

a) Because $a \ll d$, we can treat the wire as if it is a carrier of charge of linear density λ.

Use the method of images to account for the induced charge on the surface of the conducting sheet, so imagine linear charge density $-\lambda$ a distance d past the conducting sheet.



Then, if we choose the electrostatic potential, φ, to be zero on the sheet, along the line passing through the charges,

$$\Delta\varphi = -\int_0^{d-a} dx\; E(x)$$

$$E(x) = \frac{-\lambda}{2\pi\varepsilon_0}\left(\frac{1}{-x+d} + \frac{1}{x+d}\right)$$

$$\text{So } \Delta\varphi = \frac{\lambda}{2\pi\varepsilon_0}\left(\ln(x+d) - \ln(d-x)\right)\Big|_0^{d-a}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{2d-a}{d}\right) + \ln\left(\frac{d}{a}\right)\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2d-a}{a}\right)$$

The capacitance per unit length is the charge per unit length$/|\Delta\varphi|$

$$\frac{C}{L} = \lambda/|\Delta\varphi| = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2d-a}{a}\right)}$$

b) The magnitude of the electric field from the wire at $+d$ at the surace of the plane is

$$|E_+(y)| = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{\sqrt{d^2+y^2}}$$

The component $\perp$ to the plane is the above multiplied by $d/\sqrt{d^2+y^2}$.

The components $\parallel$ to the plane from the wire and its image cancel of course & the $\perp$ component is doubled:

$$|E| = \frac{\lambda}{\pi\varepsilon_0}\frac{d}{d^2+y^2} \;\rightarrow\; E(y) = -\frac{\lambda}{\pi\varepsilon_0}\frac{d}{d^2+y^2}\,\hat{x}$$

Then, the charge density is $\sigma = E\varepsilon_0$

So $\sigma(y) = \dfrac{-\lambda d}{\pi(d^2+y^2)}$

PRÍKLAD 3: overenie, že $\int_{-\infty}^{\infty} dy\, \sigma(x,y) = -\lambda$

(premyslieť prečo?)

$$-\int_{-\infty}^{\infty} dy \frac{\lambda d}{\pi(d^2+y^2)} = -\frac{\lambda}{\pi}\int_{-\infty}^{\infty}\frac{dy}{d}\frac{1}{1+\frac{y^2}{d^2}} = -\frac{\lambda}{\pi}\arctan\frac{y}{d}\Big|_{-\infty}^{\infty} =$$

$$= -\frac{\lambda}{\pi}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = \boxed{-\lambda}$$

Príklad 4

First, let us define some directions: x is the incoming direction, and y is the direction transverse to it. Imagine the test mass is incoming along the x-axis, and the mass M is sitting at $x = 0, y = b$. Let us denote the velocity components by (v_x, v_y). The deflection angle $\alpha = v_y/v_x = v_y/\sqrt{v_0^2 - v_y^2} \sim v_y/v_0 \times (1 + [v_y/v_0]^2/2 + \ldots)$, where v_y and v_x are evaluated in the far future. Small deflection is equivalent to small v_y/v_0, and in that limit α is well approximated by v_y/v_0 (the correction is order α^3). Our task is therefore to find v_y in the far future. Let us define an angle θ such that $b\tan\theta$ equals the x component of the separation between the test mass and M, and $b/\cos\theta$ equals the actual separation. (In the limit of small deflection, the y component of the separation between test mass and M is always going to be roughly b.) The equation of motion in the y direction is $\dot{v}_y = \cos\theta GM/(b/\cos\theta)^2$. Note that $v_x \sim v_0$ (this is actually not so obvious, but can be seen by noting that $v_x \sim v(1-[v_y/v]^2/2+\ldots)$, and $v_y/v \ll 1$, and v and v_0 should differ from each other by no more than $(v - v_0)/v_0 < GM/(bv_0^2)$ which as it turns out is a small quantity of order about α). Note also that $v_x = d(b\tan\theta)/dt = b\dot{\theta}/\cos^2\theta$. We therefore have $\dot{\theta} \sim (v_0/b)\cos^2\theta$. Putting this into the $\dot{v}_y$ equation, we have $dv_y/d\theta = (GM/bv_0)\cos\theta$. Integrating θ from $-\pi/2$ to $\pi/2$, we find $v_y = 2GM/(bv_0)$ in the far future, and therefore the deflection angle is $2GM/(bv_0^2)$. Incidentally, if one blindly applies this to photons, the deflection angle is off from the GR result by a factor of exactly 2.

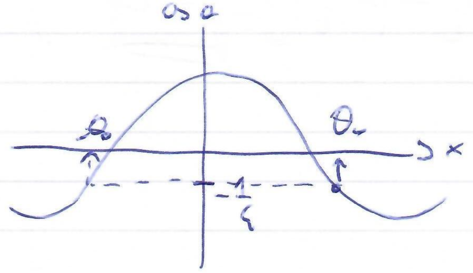
PRÍKLAD 4:

riešenie pre hyperbolu v tvare

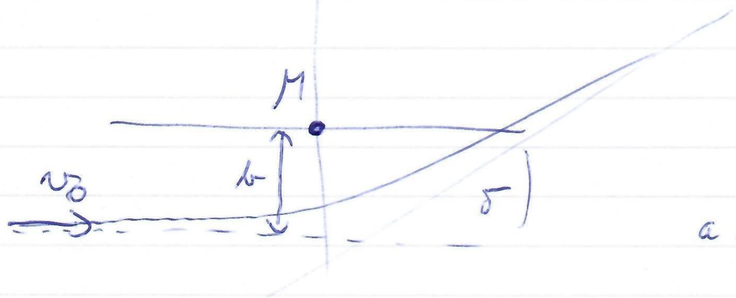
$$r = \frac{r_0}{1+\varepsilon\cos\theta} \quad (1) \qquad \text{kde} \quad \varepsilon = \sqrt{1+\frac{2EL^2}{m^3k^2}} > 1$$

kde $k = GM$

má zmysel iba keď $\cos\theta < -\frac{1}{\varepsilon}$



výsledok (1) teda dáva zmysel pre rozsah uhlov $2\theta_0 = \Delta\theta$



z obrázku $\Delta\theta = \pi + \delta$

toto hľadáme

a teda $\boxed{\delta = 2\theta_0 - \pi}$

$$E = \frac{1}{2} m v_0^2, \quad L = m v_0 b \quad \Rightarrow \quad \frac{2EL^2}{m^3k^2} = \frac{v_0^4 b^2}{G^2M^2}$$

v limite $v_0 \gg 1$ máme

$$\frac{1}{\varepsilon} \approx \frac{GM}{v_0^2 b}$$

keď $\theta_0 \approx \frac{\pi}{2} \Rightarrow \cos\theta_0 \approx \frac{\pi}{2} - \theta_0$ a teda $\theta_0 = \frac{\pi}{2} + \frac{GM}{v_0^2 b}$

dostávame $\boxed{\delta = \frac{2GM}{v_0^2 b}}$

Príklad 5

Let r be the distance of R_1 and R_2 to B. Given $r \gg \lambda$.

Let amplitude of waves be E_0

At R_1: $E_{10} = E_0 e^{ik(r-\lambda/2)} + E_0 e^{ikr}$

$+ E_0 e^{ik(r+\lambda/2)} + E_0 e^{ik\sqrt{r^2+\lambda^2/4}}$

At R_2: $E_{20} = E_0 e^{ikr} + E_0 e^{ik(r+\lambda/2)}$

$+ 2E_0 e^{ik\sqrt{r^2+\lambda^2/4}}$

But $k\lambda = 2\pi \Rightarrow e^{ik\lambda/2} = \underline{-1}$

For $r \gg \lambda$, $\sqrt{r^2+\lambda^2/4} \approx r \Rightarrow e^{ik\sqrt{r^2+\lambda^2/4}} \to e^{ikr}$

Plugging in

$E_{10} \approx 0$ and $E_{20} \approx 2E_0 e^{ikr}$

Intensity $I \propto |E|^2$

$I_1 = 0$ and $I_2 = 4E_0^2$

R_2 picks up greater signal.

(b) If B is turned off

$E_1 \approx -E_0 e^{ikr}$ $E_2 \approx E_0 e^{ikr}$

$I_1 = I_2 \sim E_0^2$

Two receivers pick up same intensity

(c) If source D is turned off

$E_1 \approx -E_0 e^{ikr}$ $E_2 \approx 3E_0 e^{ikr}$

$I_1 \sim E_0^2$ $I_2 \sim 9E_0^2$

R_2 picks up nine times greater signal.

(d) R_2 is the only receiver sensitive to the B and D source conditions.