

METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto20 – Príklady 2

VZOROVÉ RIEŠENIA

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Príklad 1

28. Kľúčovým je uvedenie si, že nás nezaujíma priebeh pohybu telieska, ale iba počiatočný a koncový stav a tiež fakt, že v gravitačnom poli sa celková energia telesa zachováva. Preto je zákon zachovania energie presne to, čo potrebujeme. Tak si ho pre našu situáciu zapíšeme:

$$-\kappa \frac{M_1 m}{R_1} - \kappa \frac{M_2 m}{2R_1 + R_2} = \frac{1}{2} m v^2 - \kappa \frac{M_2 m}{R_2} - \kappa \frac{M_1 m}{R_1 + 2R_2}$$

Odtiaľto už ľahko dostaneme hľadanú rýchlosť telieska:

$$v = 2 \sqrt{\kappa \left(M_2 \frac{R_1}{R_2(2R_1 + R_2)} - M_1 \frac{R_2}{R_1(R_1 + 2R_2)} \right)}$$

Príklad 2

Príklad 3

$$E = 0 = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + mgy$$

$$y = -ax^n \Rightarrow \dot{y} = -nax^{n-1} \dot{x}$$

$$\Rightarrow \dot{x}^2 = \frac{2gax^n}{1+n^2a^2x^{2(n-1)}}$$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$$2\cancel{\dot{x}} \ddot{x} = \frac{\partial}{\partial t} \left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}} \right) = \cancel{\dot{x}} \frac{\partial}{\partial x} \left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}} \right) = 0$$

$$\Rightarrow \frac{nx^{n-1}}{1+n^2a^2x^{2(n-1)}} - \frac{2(n-1)n^2a^2x^{3n-3}}{(1+n^2a^2x^{2(n-1)})^2} = 0$$

$$nx^{n-1} + n^3a^2x^{3n-3} - 2(n-1)n^2a^2x^{3n-3} = 0$$

$$\Rightarrow a^2x^{2n-2} = \frac{1}{n(n-2)}. \text{ Real finite solution only for } n > 2.$$

Príklad 4

(a) upper and lower branches are equivalent, so voltage across central capacitor is zero - throw it away.

$$C_a = 2 \left(\frac{1}{C} + \frac{1}{2C} \right)^{-1} = \frac{4}{3} C$$

(b) From symmetry, voltage drops on outside C -capacitors are the same and we'll denote them U_1 , and similarly for $2C$ capacitors, U_2 , with condition $U_1 + U_2 = V$ - overall voltage drop. Voltage drop across middle one is $U_2 - U_1$. Write system of equations for charges on the capacitors, $q_1 = CU_1$, $q_2 = 2CU_2$ and $q_{center} = q_1 - q_2 = C(U_2 - U_1)$ and solve this system for $U_{1,2}$ and $q_{1,2}$. The overall capacitance is

$$C_b = \frac{q_1 + q_2}{V} = \frac{7}{5}C$$

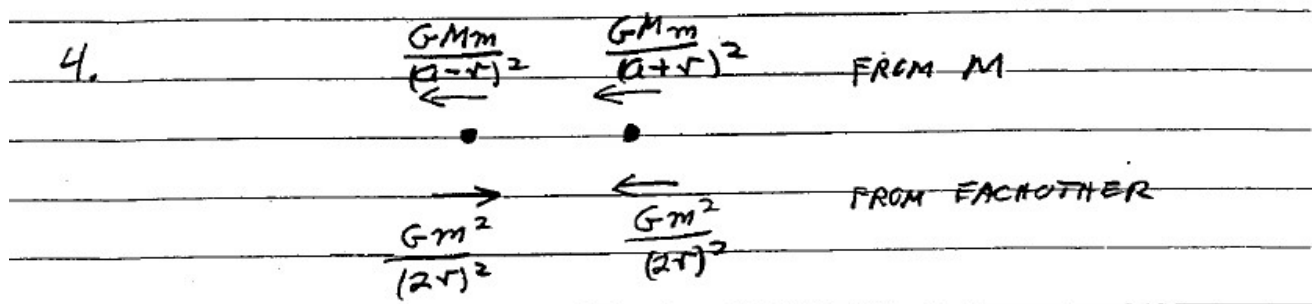
(c) make a vertical cut after first vertical capacitor - resulting infinite chain is the same as the original

$$\frac{1}{C_c} = \frac{1}{C} + \frac{1}{C + C_c} \Rightarrow C_c = \frac{\sqrt{5} - 1}{2}C$$

(d) the points with the same voltages can be connected - this results in equivalent scheme: (3 parallel)-(6 parallel)-(3 parallel)

$$\frac{1}{C_d} = \frac{1}{3C} + \frac{1}{6C} + \frac{1}{3C} = \frac{5}{6C} \Rightarrow C_d = \frac{6}{5}C$$

Príklad 5



FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE LESS THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{GMm}{4r^2}$$

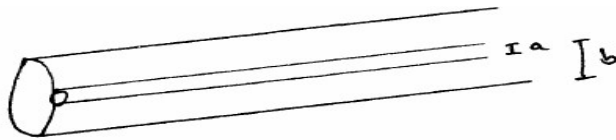
$$\frac{M}{(a^2-r^2)^2} (a+r)^2 - \frac{M}{(a^2-r^2)^2} (a-r)^2 < \frac{m}{2r^2}$$

$$4ar \frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3} \pi r^3 \rho$$

$$\frac{6}{\pi} \frac{M}{a^3} < \rho$$

Příklad 6



magnetic fields using Ampère's Law

$$r > b \quad \vec{B} = 0$$

$$a < r < b \quad \oint \vec{B} d\vec{l} = \mu_0 I \Rightarrow 2\pi r B = \mu_0 I$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

$$r < a$$

$$2\pi r B = \mu_0 I \frac{r^2}{a^2} \Rightarrow \vec{B} = \frac{\mu_0 I r}{2\pi a^2} \hat{\phi}$$

$$W = \frac{1}{2\mu_0} \iiint B^2 dV, \text{ so per unit length}$$

$$\frac{W}{l} = \frac{\mu_0}{2} \frac{I^2}{(2\pi)^2} 2\pi \left\{ \int_0^a \frac{s^2}{a^4} s ds + \int_a^b \frac{1}{s^2} s ds \right\}$$

$$= \frac{\mu_0}{4\pi} I^2 \left\{ \left[\frac{1}{4} \frac{s^4}{a^4} \right]_0^a + \left[\ln s \right]_a^b \right\}$$

$$= \frac{\mu_0}{16\pi} I^2 + \frac{\mu_0}{4\pi} \ln \frac{b}{a} I^2$$

Now use $W = \frac{1}{2} L I^2$, so

$$\frac{L}{l} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

↑
vanishes for
cylindrical shell