METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto20 – Príklady 3

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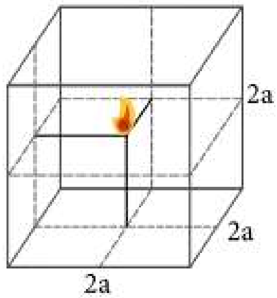
Príklad 1

Predstavme si, že máme lampáš tvaru kocky so stranou dĺžky $2a$. Ak by sme umiestnili sviečku presne do stredu, každá zo šiestich stien by bola osvetľovaná rovnako. To znamená, že ak by boli jeho vrchná a spodná stena nepriehľadné, lampáš by osvetľoval $4/6 = 2/3$ priestoru.



Keď sa však pozrieme na Adamov lampáš, vytvoríme ho z nášho tak, že celú spodnú polovicu nahradíme nepriehľadnou podstavou rozmeru $2a \times 2a$. Takže všetko svetlo, ktoré by mohlo vychádzať von spodnou polovicou nášho lampáša, z Adamovho vychádzať nebude. Po tejto úvahe je už zjavné, že Adamov lampáš bude osvetľovať presne $1/3$ priestoru.

Príklad 2

Príklad 3

We need to find force acting on liquid in one quarter of the pipe. For a small segment of the pipe,

$$dF = \frac{dm\, v^2}{R} = \rho S d\phi\, v^2$$

and it is pointing towards the center, providing centripetal acceleration to the liquid. Projection of the total force on the horizontal axis is $F_x = \int_0^{\pi/2} dF \cos\phi$, and the answer is $T = F_x$,

$$\boxed{T = \rho S v^2}$$

This we can get from more sophisticated hydrodynamics as well, by considering momentum flux equation

$$\frac{\partial}{\partial t}\rho v_i = -\boldsymbol{\nabla}_j \Pi_{ij} + f_i$$

where $\Pi_{ij} = \delta_{ij} p + \rho v_i v_j$ is momentum density tensor, and f_i is external force density. Considering left quarter of the pipe, for steady flow there is no momentum change in this volume, and thus we must have for the external force acting on the volume

$$F_i = \int dV\, f_i = \int dV\, \boldsymbol{\nabla}_j \Pi_{ij} = \oint dS_j \Pi_{ij} = \oint \rho v_i (\mathbf{v} d\mathbf{S})$$

where we used Gauss theorem and the fact that the pressure is uniform. The surface integral is not zero only through opening parts of the pipe, since on the sides $\mathbf{v} \perp d\mathbf{S}$. For x-component of the force we integrate only over the vertical cross-section at the top of the semi-circle, to immediately get

$$T = F_x = \rho v (vS) = \rho S v^2$$

Príklad 4

ANGULAR MOMENTUM CONSERVATION:

INITIALLY $L = |\vec{r} \times \vec{p}| = mvr|\sin\theta| = mvb$

AT PERIGEE $L = mv_p r_p$

$\therefore v_p = vb/r_p$

ENERGY CONSERVATION: $KE + \frac{GMm}{r_p} \simeq 0$

$\therefore \frac{1}{2}mv_p^2 = \frac{GMm}{r_p}$

SUBSTITUTE FOR v_p: $\frac{v^2b^2}{2r_p^2} = \frac{GM}{r_p}$

HENCE: $r_p = \frac{v^2b^2}{2GM}$

Príklad 5

One can express the net radiative transfer, I_{net}, in terms of the left and right portions of the system.

$$I_R = \epsilon_L \sigma T_L^4 + (1 - \epsilon_L) I_L$$

$$I_L = \epsilon_R \sigma T_R^4 + (1 - \epsilon_R) I_R$$

$$I_{\text{net}} = I_R - I_L$$

By looking at the quantity $\epsilon_R I_R - \epsilon_L I_L$ and isolating $I_R - I_L$, one finds...

$$\boxed{I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{\epsilon_R + \epsilon_L - \epsilon_L \epsilon_R}}$$

For the infinite series approach, consider the contribution from each sequence. That is.

$$I'_R = \epsilon_R \epsilon_L \sigma T_L^4 (1 + (1 - \epsilon_R)(1 - \epsilon_L) + ((1 - \epsilon_R)(1 - \epsilon_L))^2 + ...)$$

and the same for I'_L. The infinite series term is just the geometric series with a value $(1 - (1 - \epsilon_R)(1 - \epsilon_L))^{-1}$. Therefore, the net effect is

$$I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{1 - (1 - \epsilon_R)(1 - \epsilon_L)}$$

Which is the same as above.