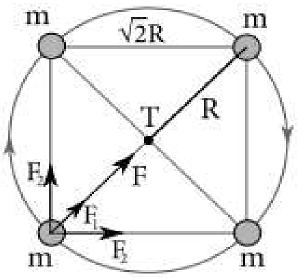
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto20 – Príklady 4

VZOROVÉ RIEŠENIA

Cvičenie 26.3.2020

Príklad 1

Planéty na seba navzájom pôsobia gravitačnými silami. Situácia je zjavne úplne stredovo súmerná, takže môžeme spočítať, aká sila pôsobí na ľubovoľnú z nich.



Protiľahlá planéta sa nachádza vo vzdialenosti $2R$, preto pôsobí silou veľkosti $F_1 = \frac{Gm^2}{4R^2}$. Priľahlé planéty sa nachádzajú vo vzdialenosti $\sqrt{2}R$, preto budú pôsobiť silami veľkosti $F_2 = \frac{Gm^2}{2R^2}$. Zo symetrie úlohy je zrejmé, že výslednica síl od dvoch priľahlých planét bude smerovať do stredu, preto si ich rozložme do dostredného smeru a smeru naň kolmého. Kolmé zložky sa vybijú a prežijú iba dostredné zložky $\frac{1}{\sqrt{2}}F_2$.

Výsledná sila pôsobiaca na planétu teda bude

$$F = F_1 + 2 \cdot \frac{1}{\sqrt{2}} F_2 = \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) \frac{Gm^2}{R^2}.$$

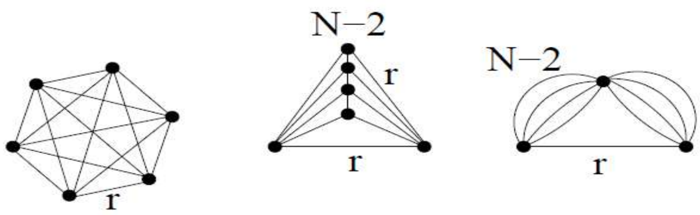
Táto sila spôsobuje pohyb po kružnici, čiže je dostredivou silou $F = m\omega^2 R$. Z rovnosti dostredivej a výslednej gravitačnej sily dostávame $\omega = \sqrt{\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) \frac{Gm}{R^3}}$. Z definície uhlovej rýchlosti $\omega = \frac{2\pi}{T}$ dopočítame periódu obehu planét

$$T = \frac{4\pi}{\sqrt{2\sqrt{2}+1}} \sqrt{\frac{R^3}{Gm}}.$$

Príklad 2

Príklad 3

If the number of point is N, then we have two points (IN and OUT) that are connected directly. Then we also have $N-2$ points that are connected to the IN and OUT points and all interconnected among themselves. Since all resistors are the same, all the $N-2$ points have same potential, and can be connected into one point. In other words, there is no current flowing between these middle points due to symmetry.



So our equivalent electric configuration is ($N-2$ parallel resistors in series with $N-2$ parallel resistors) in parallel with one resistor.

If the number of points is N the resistance between any two points is then

$$\boxed{R = \left(\frac{1}{r} + \frac{N-2}{2r}\right)^{-1} = \frac{2}{N} r}$$

Príklad 4

The linearity of Maxwell's equations allows us to find the magnetic field as a sum of two magnetic fields produced by two currents: A current with density

$$j_b = \frac{I}{\pi(b^2-a^2)} \tag{1}$$

carried by the cylinder of radius b and a current with density

$$j_a = -j_b \tag{2}$$

carried by a cylinder of radius a. The sum of these two currents gives the current distribution in the considered structure. From Ampere's circuital law $\oint \mathbf{H} \cdot d\mathbf{l} = \frac{4\pi}{c} \int \mathbf{j} \cdot d\mathbf{S}$ one finds that the current carried by the cylinder of radius b produces a magnetic field at the center of the hole

$$H_b = \frac{2Id}{c(b^2-a^2)}, \tag{3}$$

while the current carried by the cylinder of radius a produces no magnetic field at the center of the hole, $H_a = 0$. Therefore, the magnetic field at the center of the hole is

$$H = \frac{2Id}{c(b^2-a^2)}. \tag{4}$$

Príklad 5

$$[H,F_1] = \sum_j \left\{ \frac{\partial H}{\partial q_j}\frac{\partial F_1}{\partial p_j} - \frac{\partial H}{\partial p_j}\frac{\partial F_1}{\partial q_j} \right\}$$

$$= \left\{ (p_1 - 2aq_1)\frac{1}{q_2} - q_1\left(\frac{-a}{q_2}\right) + (-p_2 + 2bq_2)\cdot 0 - (-q_2)\left(\frac{p_1 - aq_1}{-q_2^2}\right) \right\}$$

$$= 0$$

$$[H,F_2] = \sum_j \left(\frac{\partial H}{\partial q_j}\frac{\partial F_2}{\partial p_j} - \frac{\partial H}{\partial p_j}\frac{\partial F_2}{\partial q_j} \right)$$

$$= \left\{ (p_1 - 2aq_1)\cdot 0 - q_1 q_2 + (-p_2 + 2bq_2)\cdot 0 - (-q_2)q_1 \right\}$$

$$= 0$$

$$[F_1,F_2] = \sum_j \left(\frac{\partial F_1}{\partial q_j}\frac{\partial F_2}{\partial p_j} - \frac{\partial F_1}{\partial p_j}\frac{\partial F_2}{\partial q_j} \right)$$

$$= \left\{ -\frac{a}{q_2}\cdot 0 - \frac{1}{q_2} q_2 + \left(\frac{p_1 - aq_1}{-q_2^2}\right)\cdot 0 - 0\cdot q_1 \right\}$$

$$= -1$$ $\therefore$ NO ADDITIONAL CONSTANTS.