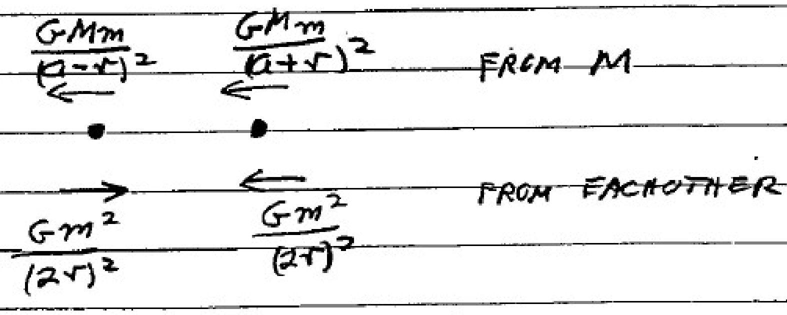
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto20 – Príklady 4

VZOROVÉ RIEŠENIA

Cvičenie 27.4.2020

Príklad 1

4.



FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE <u>LESS</u> THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{G\cancel{Mm}\,m^2}{4r^2}$$

$$\frac{M}{(a^2-r^2)^2}(a+r)^2 - \frac{M}{(a^2-r^2)^2}(a-r)^2 < \frac{m}{2r^2}$$

$$4ar\,\frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\underline{\frac{6}{\pi}\frac{M}{a^3} < \rho}$$

Príklad 2

the relevant part of the ideal gas partition function for the unmixed and mixed cases are

unmixed: $Z = Z_1 Z_2$

$$Z_1 \sim \frac{V^{N_1}}{N_1!} \frac{V^{M_1}}{M_1!} \qquad Z_2 = \frac{V^{N_2}}{N_2!} \frac{V^{M_2}}{M_2!}$$

$$S \sim -\frac{\partial F}{\partial T} \simeq K N_1 \ln \frac{V}{(N_1 \lambda_\alpha^3)} + K M_1 \ln \frac{V}{(M_1 \lambda_\beta^3)}$$

$$+ K N_2 \ln \frac{V}{(N_2 \lambda_\alpha^3)} + K M_2 \ln \frac{V}{(M_2 \lambda_\beta^3)}$$

after mixing (volume is now $2V$), where λ_α and λ_β are the thermal wavelengths for the gases α and β, $\lambda_\alpha = \text{const} \cdot \frac{\hbar}{\sqrt{2 m_\alpha k_B T}}$

$$S' = K (N_1 + N_2) \ln \frac{2V}{(N_1 + N_2)}$$

$$+ K (M_1 + M_2) \ln \frac{2V}{(M_1 + M_2)}$$

so

$$\Delta S = S' - S = K \left[N_1 \ln \frac{2N_1}{(N_1 + N_2)} + N_2 \ln \frac{2N_2}{(N_1 + N_2)} + M_1 \ln \frac{2M_1}{(M_1 + M_2)} + M_2 \ln \frac{2M_2}{(M_1 + M_2)} \right]$$

if $N_1 = N_2 = \frac{1}{2}(N_1 + N_2)$ $\Delta S = 0$: if $M_1 = N_2 = 0$ $\Delta S = 2 N_1 K \ln 2$

Príklad 3

SOLUTION: Find $\vec{B} = \widehat{\phi} 2I/rc$ and then $\vec{A} = -\widehat{z}(2I/c)\ln(r/a)$, so the electrons have

$$L = -mc^2\sqrt{1 - v^2/c^2} + (2I|e|/c^2)v_z \ln(r/a).$$

The energy and p_z are conserved:

$$p_z = \frac{\partial L}{\partial v_z} = \gamma m v_z + (2I|e|/c^2)\ln(r/a) = \gamma_0 m v_0.$$

$$H = \gamma mc^2 = \gamma_0 mc^2$$

with $\gamma = 1/\sqrt{1 - v^2/c^2}$ and $\gamma_0 \equiv 1/\sqrt{1 - v_0^2/c^2}$. So $\gamma = \gamma_0$ and r_{max} is where $\dot{r} = 0$, which means that $v_z = -v_0$ (half-period of cyclotron rotation), which gives

$$r_{max} = a\exp(\gamma_0 m v_0 c^2/I|e|).$$

Príklad 4

SOLUTION: For large x note that $\exp(-t - \frac{x^2}{4t})$ has a maximum at $t_0 = \frac{1}{2}x$. Set $t = \frac{1}{2}x(1 + u)$ and expand $(1 + u)^{-1}$ in the exponent to find

$$F_\nu(x) = \tfrac{1}{2}\int_{-1}^{\infty} \exp[-\tfrac{1}{2}x(1 + u + (1 - u + u^2 - u^3 + \ldots))](1 + u)^{-\nu-1}du.$$

For large x the integrand has an effective u range which is $O(1/\sqrt{x}) \ll 1$ around zero. Therefore,

$$F_\nu(x) \approx \tfrac{1}{2}e^{-x}\int_{-\infty}^{\infty} e^{-\frac{1}{2}xu^2}du = \sqrt{\frac{\pi}{2x}}e^{-x},$$

independent of ν. (The function in question is the modified Bessel function $K_\nu(x)$.)