METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto19 – Príklady 2

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Príklad 1

Extensible molecule: Consider a long molecule which is composed of N chemical units ('monomers'), each of which can be in one of two states, of different lengths a and b, with $b > a$. The whole molecule therefore can be between Na and Nb in length. The energy of a monomer in the longer state is ϵ larger than a monomer in the shorter state. You may consider the thermodynamic limit $N >> 1$ to simplify the calculations.

(a) Calculate the equilibrium length of the entire molecule as a function of temperature T.

(b) Calculate the root-mean-square fluctuation in the length of the entire molecule as a function of temperature T.

(c) Now, suppose that the molecule is forced to be a *fixed* length $L(Na < L < Nb)$, so that $(L - Na)/(b - a)$ of its monomers are in the stretched (length b) state. Find the internal energy $E(N, L)$ and the entropy $S(N, T, L)$.

(d) From (c) calculate the Helmholtz free energy $F(N, T, L)$, and finally the force needed to extend the molecule to to length L at fixed temperature T.

Príklad 2

Electron states in graphene are described by the two-component Schrödinger equation

$$\begin{bmatrix} -\varepsilon & v(\pi_x - i\pi_y) \\ v(\pi_x + i\pi_y) & -\varepsilon \end{bmatrix} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad v \approx \frac{c}{300}.$$

When a uniform magnetic field $-\hat{\mathbf{z}}B$ is present, the generalized momentum operators are

$$\pi_x = -i\hbar\partial_x + \frac{e}{c}A_x, \quad \pi_y = -i\hbar\partial_y + \frac{e}{c}A_y,$$

and the vector potential obeys $\partial_x A_y - \partial_y A_x = -B$, as usual.

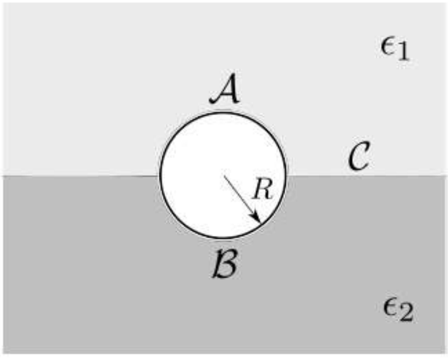
(a) Verify that that $[\pi_x, \pi_y] = i\hbar^2/\ell^2$, where $\ell = \sqrt{\hbar c/eB}$.

(b) Construct an operator a and its Hermitian conjugate $a^\dagger$ from π_x and $i\pi_y$ such that $[a, a^\dagger] = 1$.

(c) Rewrite the Schrödinger equation above in terms of a and $a^\dagger$ and convert it into two independent equations for ψ_A and ψ_B.

(d) Using an analogy to the harmonic oscillator problem, find the quantized energy levels ε_n (known as Landau levels). Can ε_n be negative?

Príklad 3



The center of a conducting sphere of radius R is located on the flat boundary between two dielectrics each filling half of the whole space outside the sphere. The dielectric permittivities are ε_1 and ε_2. The conducting sphere is held at potential V. Consider the space outside of the conducting sphere.

(a) Show that the potential $\Phi = VR/r$ satisfies the required boundary conditions on the plane $\mathcal{C}$ separating dielectrics as well as on the sphere.

(b) Find the free charge density σ on the surface of the conducting sphere and the total amount of free charge Q on it.

(c) Find the bound charge densities σ_b on the spherical boundaries $\mathcal{A}$ and $\mathcal{B}$ of the dielectrics.

(d) Find the bound charge density σ_b on the flat boundary $\mathcal{C}$ between the dielectrics.

Príklad 4

You are mountain climbing on a conical peak described by the equation $z = -\sqrt{x^2 + y^2}$. There is a storm coming and you need to take refuge quickly. What is the equation of the shortest path to the refuge at position $(-1, 0, -1)$ if you are now located at $(1, 0, -1)$.