

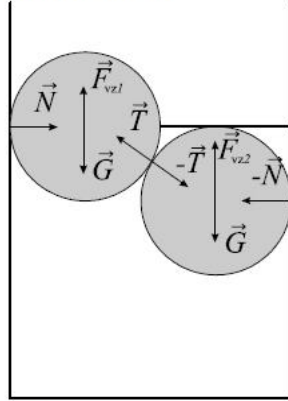
## METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima20 – Príklady 1

### VZOROVÉ RIEŠENIA

Cvičenie 2.10.2020

#### Príklad 1

15. Na každé brvno pôsobí sila zo strany nádoby  $\vec{N}$ , tiažová sila  $\vec{G}$ , vztlaková sila  $\vec{F}_{vz}$  a sila zo strany druhého brvna  $\vec{T}$ .



Smer sily  $\vec{T}$  určíme z pravouhlého trojuholníka, ktorého prepona je spojnicou stredov kružníc a jeho jedna odvesna je kolmá na hladinu. Prepona má dĺžku  $2r$ , odvesna dĺžku  $r$ , teda sila  $\vec{T}$  zvierá s horizontálou uhol  $30^\circ$ . V rovnováhe je súčet síl nulový. Pre ľavé brvno dostávame rovnice

$$\begin{aligned} V\rho_0 - mg - T \sin 30^\circ &= 0 \\ N - T \cos 30^\circ &= 0 \end{aligned}$$

Pre pravé brvno

$$\frac{V}{2} \rho_0 g - mg + T \sin 30^\circ = 0$$

druhá rovnica je identická ako pre ľavé brvno. Riešením týchto rovníc dostávame  $T = mg/\sqrt{3}$ .

Příklad 2

Let  $r$  be the distance of  $R_1$  and  $R_2$  to  $B$ . Given  $r \gg \lambda$ .

Let amplitude of waves be  $E_0$

$$\text{At } R_1: E_{10} = E_0 e^{ik(r-\lambda/2)} + E_0 e^{ikr} + E_0 e^{ik(r+\lambda/2)} + E_0 e^{ik\sqrt{r^2+\lambda^2/4}}$$

$$\text{At } R_2: E_{20} = E_0 e^{ikr} + E_0 e^{ik(r+\lambda/2)} + 2E_0 e^{ik\sqrt{r^2+\lambda^2/4}}$$

$$\text{But } k\lambda = 2\pi \Rightarrow e^{ik\lambda/2} = -1$$

$$\text{For } r \gg \lambda, \sqrt{r^2+\lambda^2/4} \approx r \Rightarrow e^{ik\sqrt{r^2+\lambda^2/4}} \rightarrow e^{ikr}$$

Plugging in

$$E_{10} \approx 0 \quad \text{and} \quad E_{20} \approx 2E_0 e^{ikr}$$

Intensity  $I \propto |E|^2$

$$I_1 = 0 \quad \text{and} \quad I_2 = 4E_0^2$$

$R_2$  picks up greater signal.

(b) If B is turned off

$$E_1 \approx -E_0 e^{ikr} \quad E_2 \approx E_0 e^{ikr}$$

$$I_1 = I_2 \sim E_0^2$$

Two receivers pick up same intensity

(c) If source D is turned off

$$E_1 \approx -E_0 e^{ikr} \quad E_2 \approx 3E_0 e^{ikr}$$

$$I_1 \sim E_0^2 \quad I_2 \sim 9E_0^2$$

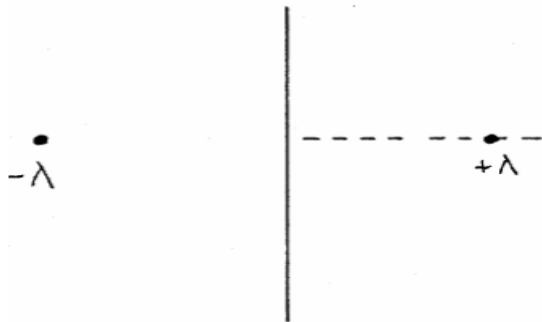
$R_2$  picks up nine times greater signal.

(d)  $R_2$  is the only receiver sensitive to the B and D source conditions.

Příklad 3

- a) Because  $a \ll d$ , we can treat the wire as if it is a carrier of charge of linear density  $\lambda$ .

Use the method of images to account for the induced charge on the surface of the conducting sheet, so imagine linear charge density  $-\lambda$  a distance  $d$  past the conducting sheet.



Then, if we choose the electrostatic potential,  $\phi$ , to be zero on the sheet, along the line passing through the charges,

$$\Delta \phi = - \int_0^{d-a} dx E(x)$$

$$E(x) = \frac{-\lambda}{2\pi\epsilon_0} \left( \frac{1}{-x+d} + \frac{1}{x+d} \right)$$

$$\begin{aligned} \text{So } \Delta \phi &= \frac{\lambda}{2\pi\epsilon_0} \left( \ln(x+d) - \ln(d-x) \right) \Big|_0^{d-a} \\ &= \frac{\lambda}{2\pi\epsilon_0} \left( \ln\left(\frac{2d-a}{d}\right) + \ln\left(\frac{d}{a}\right) \right) = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{2d-a}{a}\right) \end{aligned}$$

The capacitance per unit length is the charge per unit length /  $|\Delta \phi|$

$$\frac{C}{L} = \frac{\lambda}{|\Delta \phi|} = \frac{2\pi\epsilon_0}{\ln\left(\frac{2d-a}{a}\right)}$$

- b) The magnitude of the electric field from the wire at  $+d$  at the surface of the plane is

$$|E_+(y)| = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{\sqrt{d^2+y^2}}$$

The component  $\perp$  to the plane is the above multiplied by  $\frac{d}{\sqrt{d^2+y^2}}$ .

The components  $\parallel$  to the plane from the wire and its image cancel of course & the  $\perp$  component is doubled:

$$|E| = \frac{\lambda}{\pi\epsilon_0} \frac{d}{d^2+y^2} \rightarrow E(y) = -\frac{\lambda}{\pi\epsilon_0} \frac{d}{d^2+y^2} \hat{x}$$

Then, the charge density is  $\sigma = E \epsilon_0$

$$\text{So } \sigma(y) = \frac{-\lambda d}{\pi(d^2 + y^2)}$$

PR/UKUO 3: overenie, že  $\int_{-\infty}^{\infty} dy \sigma(y) = -\lambda$   
 (premysliet prečo?)

$$\begin{aligned} -\int_{-\infty}^{\infty} dy \frac{\lambda d}{\pi(d^2 + y^2)} &= -\frac{\lambda}{\pi} \int_{-\infty}^{\infty} \frac{dy}{d} \frac{1}{1 + \frac{y^2}{d^2}} = -\frac{\lambda}{\pi} \arctan \frac{y}{d} \Big|_{-\infty}^{\infty} = \\ &= -\frac{\lambda}{\pi} \left[ \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right] = \boxed{-\lambda} \end{aligned}$$

#### Příklad 4

First, let us define some directions:  $x$  is the incoming direction, and  $y$  is the direction transverse to it. Imagine the test mass is incoming along the  $x$ -axis, and the mass  $M$  is sitting at  $x = 0, y = b$ . Let us denote the velocity components by  $(v_x, v_y)$ . The deflection angle  $\alpha = v_y/v_x = v_y/\sqrt{v_0^2 - v_y^2} \sim v_y/v_0 \times (1 + [v_y/v_0]^2/2 + \dots)$ , where  $v_y$  and  $v_x$  are evaluated in the far future. Small deflection is equivalent to small  $v_y/v_0$ , and in that limit  $\alpha$  is well approximated by  $v_y/v_0$  (the correction is order  $\alpha^3$ ). Our task is therefore to find  $v_y$  in the far future. Let us define an angle  $\theta$  such that  $b \tan \theta$  equals the  $x$  component of the separation between the test mass and  $M$ , and  $b/\cos \theta$  equals the actual separation. (In the limit of small deflection, the  $y$  component of the separation between test mass and  $M$  is always going to be roughly  $b$ .) The equation of motion in the  $y$  direction is  $\dot{v}_y = \cos \theta GM / (b/\cos \theta)^2$ . Note that  $v_x \sim v_0$  (this is actually not so obvious, but can be seen by noting that  $v_x \sim v(1 - [v_y/v]^2/2 + \dots)$ , and  $v_y/v \ll 1$ , and  $v$  and  $v_0$  should differ from each other by no more than  $(v - v_0)/v_0 < GM/(bv_0^2)$  which as it turns out is a small quantity of order about  $\alpha$ ). Note also that  $v_x = d(b \tan \theta)/dt = b \dot{\theta} / \cos^2 \theta$ . We therefore have  $\dot{\theta} \sim (v_0/b) \cos^2 \theta$ . Putting this into the  $\dot{v}_y$  equation, we have  $dv_y/d\theta = (GM/bv_0) \cos \theta$ . Integrating  $\theta$  from  $-\pi/2$  to  $\pi/2$ , we find  $v_y = 2GM/(bv_0)$  in the far future, and therefore the deflection angle is  $2GM/(bv_0^2)$ . Incidentally, if one blindly applies this to photons, the deflection angle is off from the GR result by a factor of exactly 2.

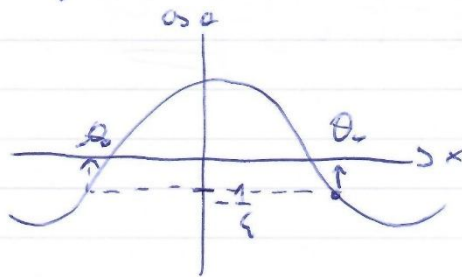
PRÍKLAD 48

trajektorie pre hyperbolu v hviezd

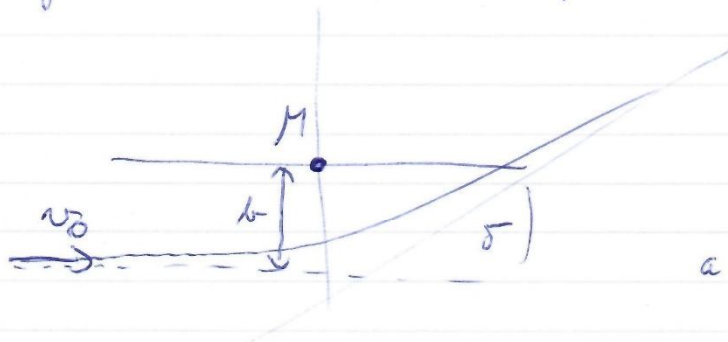
$$r = \frac{r_0}{1 + \epsilon \cos \theta} \quad (1) \quad \text{kde} \quad \epsilon = \sqrt{1 + \frac{2EL^2}{m^3 k^2}} > 1$$

kde  $k = GM$

naše trajektorie iba keď  $\cos \theta < -\frac{1}{\epsilon}$



výsledok (1) keď dáva trajektorie pre rozbíhajúceho  $2\theta_0 = \Delta\theta$



z obrázku  $\Delta\theta = \pi + \delta$

ako kvadrans

a keď  $\delta = 2\theta_0 - \pi$

$$E = \frac{1}{2} m v_0^2 \Rightarrow \frac{2EL^2}{m^3 k^2} = \frac{v_0^4 b^2}{G^2 M^2} \quad \text{v limite } v_0 \gg 1 \text{ máme}$$

$$L = m v_0 b \quad \frac{1}{\epsilon} \approx \frac{GM}{E v_0^2 b}$$

keď  $\theta_0 \approx \frac{\pi}{2} \Rightarrow \cos \theta_0 \approx \frac{\pi}{2} - \theta_0$  a keď  $\theta_0 = \frac{\pi}{2} + \frac{GM}{v_0^2 b}$

kvadrans

$$\delta = \frac{2GM}{v_0^2 b}$$