METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima20 – Príklady 2

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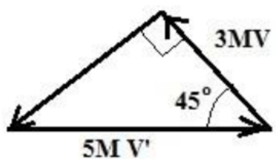
Príklad 1



METHOD #1: Vector Addition: Since $\vec{p}_{3M} + \vec{p}_{4M} + \vec{p}_{5M} = 0$ as the object was initially at rest, $\vec{p}_{3M} + \vec{p}_{4M} = -\vec{p}_{5M}$. Since p_{3M} and p_{4M} form a right angle with p_{5M} as the hypotenuse, we use right-triangle trig to obtain $\cos 45^\circ = \frac{p_{3M}}{p_{5M}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{3MV}{5MV'} \Rightarrow V' = \frac{3\sqrt{2}}{5}V$

METHOD #2: Momentum components: By writing linear momentum conservation in both the x and y directions, we have $\Delta p_x = 0 \;\Rightarrow\; \Delta p_{3M_x} + \Delta p_{4M_x} + \Delta p_{5M_x} = 0 \;\Rightarrow$

$$3M\left(-V\cos 45^\circ - 0\right) + 4M\left(-V'\cos 45^\circ - 0\right) + 5M(V'' - 0) = 0$$

$$-\frac{3V}{\sqrt{2}} - -\frac{4V'}{\sqrt{2}} + 5V'' = 0 \;\Rightarrow\; 5\sqrt{2}\,V'' = 3V + 4V'$$

$$\Delta p_y = 0 \;\Rightarrow\; \Delta p_{3M_y} + \Delta p_{4M_y} + \Delta p_{5M_y} = 0 \;\Rightarrow$$

$$3M\left(-V\cos 45^\circ - 0\right) + 4M\left(-V'\cos 45^\circ - 0\right) + 5M(V'' - 0) = 0$$

$$3M\left(V\sin 45^\circ - 0\right) + 4M\left(-V'\sin 45^\circ - 0\right) + 5M(0 - 0) = 0$$

$$V' = \frac{3}{4}V$$

So, by substituting this result into the y-expression gives $5\sqrt{2}\,V'' = 3V + 3V \;\Rightarrow V'' = \frac{6}{5\sqrt{2}}V = \frac{3\sqrt{2}}{5}V$

Príklad 2

$$\underset{\sim}{E}(\underset{\sim}{r},t) = \hat{y}\,\frac{1}{2}\left[f(x-ct) + f(x+ct)\right]$$

$$\underset{\sim}{B}(\underset{\sim}{r},t) = \hat{z}\,\frac{1}{2c}\left[f(x-ct) - f(x+ct)\right]$$

Príklad 3

(a) The Lagrangian is

$$L = \tfrac{1}{2}m_1\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \tfrac{1}{2}m_2\left(\dot{\rho}^2 + \rho^2\dot{\phi}^2\right) - V(\alpha x + \beta y + \gamma z, y + a\phi).$$

(b) The Lagrangian is rendered invariant under time-independent transformations satisfying $dV = 0$ and $d\rho = 0$, i.e.

$$\alpha\,dx + \beta\,dy + \gamma\,dz = 0 \quad , \quad dy + a\,d\phi = 0 \quad , \quad d\rho = 0\,.$$

The two-parameter family may be written as

$$\begin{aligned}
x(\zeta_1,\zeta_2) &= x - \frac{\gamma}{\alpha}\zeta_1 + \frac{\beta}{\alpha}\zeta_2 \\
y(\zeta_1,\zeta_2) &= y - \zeta_2 \\
z(\zeta_1,\zeta_2) &= z + \zeta_1 \\
\phi(\zeta_1,\zeta_2) &= \phi + a^{-1}\zeta_2 \\
\rho(\zeta_1,\zeta_2) &= \rho\,.
\end{aligned}$$

The associated conserved quantities are

$$\Lambda_i = \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma} \frac{\partial q_\sigma}{\partial \zeta_i}\bigg|_{\zeta_i} ,$$

whence

$$\Lambda_1 = -\frac{\gamma}{\alpha} m_1 \dot{x} + m_1 \dot{z} \quad , \quad \Lambda_2 = \frac{\beta}{\alpha} m_1 \dot{x} - m_1 \dot{y} + a^{-1} m_2 \rho^2 \dot{\phi} .$$

(c) Since $\partial L/\partial t = 0$, the total energy is also conserved:

$$E = \tfrac{1}{2} m_1 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \tfrac{1}{2} m_2 (\dot{\rho}^2 + \rho^2 \dot{\phi}^2) + V(\alpha x + \beta y + \gamma z, y + a\phi).$$

Príklad 4

Solution : Fermat's principle $\delta \int n(\vec{r})\, d\ell = 0$.

Assuming a circular orbit of radius $R+h$, we get

$$\frac{d}{dh}\left[n(h) \cdot 2\pi (R+h) \right] = 0 \quad , \quad \frac{n'(h)}{n(h)} = -\frac{1}{R+h} ,$$

$$h = \frac{1}{2}\left(\frac{n_0}{\alpha} - R \right).$$

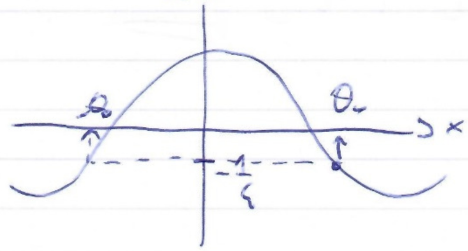
PRÍKLAD 4:

riešenie pre hyperbolu v tvare

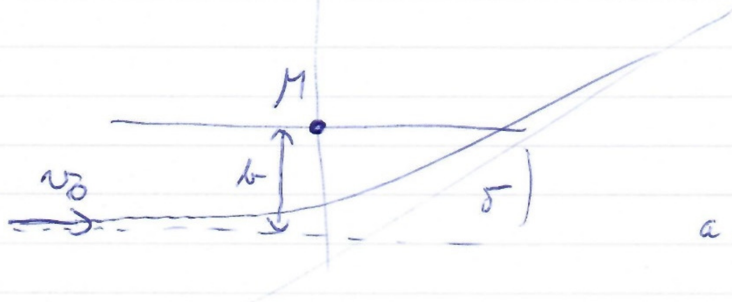
$$r = \frac{r_0}{1+\varepsilon\cos\theta} \quad (1) \qquad \text{kde} \quad \varepsilon = \sqrt{1+\frac{2EL^2}{m^3k^2}} > 1$$

kde $k = GM$

má zmysel iba keď $\cos\theta < -\frac{1}{\varepsilon}$



výsledok (1) teda dáva zmysel pre rozsah uhlov $2\theta_0 = \Delta\theta$



z obrázku $\Delta\theta = \pi + \delta$

toto hľadáme

a teda $\boxed{\delta = 2\theta_0 - \pi}$

$E = \frac{1}{2} m v_0^2$
$L = m v_0 b$ $\Rightarrow \frac{2EL^2}{m^3k^2} = \frac{v_0^4 b^2}{G^2M^2}$ v limite $v_0 \gg 1$ máme $\frac{1}{\varepsilon} \approx \frac{GM}{v_0^2 b}$

keďže $\theta_0 \approx \frac{\pi}{2} \Rightarrow \cos\theta_0 \approx \frac{\pi}{2} - \theta_0$ a teda $\theta_0 = \frac{\pi}{2} + \frac{GM}{v_0^2 b}$

dostávame $\boxed{\delta = \frac{2GM}{v_0^2 b}}$