METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima20 – Príklady 5

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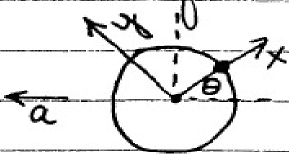
Príklad 1

a) fixed frame is ground; tire is fixed in rotating frame



$$\vec{a}_f = \vec{a}_r + \ddot{\vec{R}}_f + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2\vec{\omega}\times\vec{v}_r$$

translational + ang. acceleration

centrifugal

Coriolis

but $\ddot{\vec{R}}_f = -a\cos\theta\,\hat{x} + a\sin\theta\,\hat{y}$

$\vec{r} = r_0\hat{x}$ $\quad v_r = a_r = 0$

$\vec{\omega} = \frac{v}{r_0}\hat{z}$ $\quad \dot{\vec{\omega}} = \frac{a}{r_0}\hat{z}$

$$\therefore\ \vec{a}_f = -a\cos\theta\,\hat{x} + a\sin\theta\,\hat{y} + a\hat{y} - \frac{v^2}{r_0}\hat{x}$$

$$= -\hat{x}\left(\frac{v^2}{r_0} + a\cos\theta\right) + \hat{y}\,a(\sin\theta + 1)$$

Príklad 2

SOLUTION: The initial speed of the block is $v = 0$ and the initial speed of the board is $V = v_0$. The total momentum of the system is conserved, because the surface is frictionless. Thus, the total momentum is $P = MV + mv = Mv_0$ at all times. Now while the total momentum P is conserved, the total energy E is not, due to the friction between the board and the block. The kinetic energy of the system is given by $E = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$ and must be equal to $E_0 - W$, where $E_0 = \frac{1}{2}Mv_0^2$ is the initial kinetic energy of the system and $W = \mu mgd$ is the work done against friction for the block to slide a distance d to the left relative to the board. The minimum value of v_0 must occur when $V = v$ and $d = L$. Thus, we have two equations in the two unknowns (v_0, v):

$$(M+m)v = Mv_0$$
$$\tfrac{1}{2}(M+m)v^2 = \tfrac{1}{2}Mv_0^2 - \mu mgL\ .$$

The solution is

$$v_0 = \sqrt{2\mu gL\Big(1 + \frac{m}{M}\Big)}\ .$$

Príklad 3

Answer: In general $\nabla^2\phi = 0$ in the box gives

$$\phi(x,y,z) = \sum_{m_1,m_2} \sin\frac{m_1\pi x}{a}\sin\frac{m_2\pi y}{b}\left(A_{m_1m_2}\sinh K_{m_1m_2}z + B_{m_1m_2}\cosh K_{m_1m_2}z\right)$$

because $\phi|_{x=0,a} = \phi|_{y=0,b} = 0$. In the above

$$K^2_{m_1m_2} = \left(\frac{m_1\pi}{a}\right)^2 + \left(\frac{m_2\pi}{b}\right)^2.$$

$$\phi\Big|_{z=0} = V_1\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \Rightarrow B_{m_1,m_2} = V_1\delta_{m_1 1}\delta_{m_2 2}$$

$$\phi\Big|_{z=c} = V_2\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b} \Rightarrow \boxed{A_{22} = \frac{V_2}{\sinh K_{22}c}}$$

and $V_1\cosh K_{11}c + A_{11}\sinh K_{11}c = 0$

$$\boxed{A_{11} = -V_1\frac{\cosh K_{11}c}{\sinh K_{11}c}}$$

All other $A_{ij} = 0$

Príklad 4

SOLUTION: Find $\vec{B} = \hat{\phi}2I/rc$ and then $\vec{A} = -\hat{z}(2I/c)\ln(r/a)$, so the electrons have

$$L = -mc^2\sqrt{1 - v^2/c^2} + (2I|e|/c^2)v_z\ln(r/a).$$

The energy and p_z are conserved:

$$p_z = \frac{\partial L}{\partial v_z} = \gamma m v_z + (2I|e|/c^2)\ln(r/a) = \gamma_0 m v_0.$$

$$H = \gamma mc^2 = \gamma_0 mc^2$$

with $\gamma = 1/\sqrt{1 - v^2/c^2}$ and $\gamma_0 \equiv 1/\sqrt{1 - v_0^2/c^2}$. So $\gamma = \gamma_0$ and r_{max} is where $\dot{r} = 0$, which means that $v_z = -v_0$ (half-period of cyclotron rotation), which gives

$$r_{max} = a\exp(\gamma_0 m v_0 c^2/I|e|).$$