METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima20 – Príklady 6

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Príklad 1

4. Two blocks and two pulleys (Question from Dave, solution from Peter)

Most straightforward to use Lagrangian with constraint for fixed length of rope:

$L=\frac{1}{2}\left(m_1\dot{x}^2+m_2\dot{y}^2\right)+m_1gx$ where (referring to the diagram with the problem) x

$g(x,y)=0=x+2y+d-l$

increases downward and y increases to the right. The rope has length l and d accounts for all the rope not taken up by x and y. Also from the constraint, we have $\ddot{x}=-2\ddot{y}$. Using the Euler-Lagrange equations with undetermined multiplier gives

$$\begin{array}{c} m_1\ddot{x}-m_1g+\lambda=0 \\ m_2\ddot{y}+2\lambda=0 \end{array} \Rightarrow \begin{array}{c} -2m_1\ddot{x}+2m_1g-2\lambda=0 \\ -\frac{m_2\ddot{x}}{2}+2\lambda=0 \end{array} \Rightarrow \ddot{x}=\frac{4m_1g}{4m_1+m_2}$$

Check limits: $m_1>>m_2$, acceleration is g, : $m_1<<m_2$, acceleration goes to zero.

Príklad 2

3. Current carrying wire (Question and solution from Peter)

a) Take loop of radius r around wire and use Ampere's law:
$\oint\vec{B}\cdot d\vec{l}={}^{4\pi}\!/_{c}\,I\Rightarrow\vec{B}=B(r)\hat{\varphi}=\frac{2I}{rc}\hat{\varphi}\Rightarrow\vec{F}=q\frac{\vec{v}}{c}\times\vec{B}=\frac{2Ive}{rc^2}\hat{\rho}$ where $\hat{\rho}$ is radial direction in cylindrical coordinates.

b) If F' moves along wire with v, then particle is at rest in F' and the Lorentz force is zero. Transform force from F:
$F_\perp=\frac{dp_\perp}{dt}=\frac{dp'_\perp}{\gamma dt'-\beta\gamma dx'}=\frac{dp'_\perp/dt'}{\gamma+\beta\gamma\, dx'/dt'}=\frac{F'_\perp}{\gamma}\Rightarrow F'_{\perp=}=\gamma\frac{2Ive}{rc^2}$. Can also do problem by transforming currents and charge densities.

Príklad 3

(a) Introduce ϕ to represent rotation of the moving sphere about its center of mass. Equating the total kinetic energy with the potential energy lost gives:

$$\begin{aligned}(R+r)(1-\cos(\theta))mg &= m(r\dot{\phi})^2+\frac{2}{5}mr^2\dot{\phi}^2\\ &= \frac{7}{5}mr^2\dot{\phi}^2\\ &= \frac{7}{5}mv_{\rm cm}^2\end{aligned}$$

Thus, $v_{\rm cm}(\theta)=\sqrt{\frac{5}{7}(R+r)(1-\cos(\theta))g}$.

(b) The sphere will fly off when $mv_{\mathrm{cm}}^2/(R+r) > mg\cos(\theta)$ or

$$\frac{5}{7}(1-\cos(\theta)) \;>\; \cos(\theta)$$

or

$$\cos(\theta) = 5/13$$

(c) Start with the equation of motion obtained by equating torque and rate of change of angular momentum around the point of contact: $\frac{7}{5}mr^2\ddot{\phi} = mgr\sin\theta$. Relate θ and ϕ by computing the velocity of the moving sphere's center of mass two ways:

$$(R+r)\dot{\theta} = r\dot{\phi}$$

Combining these equations:

$$\ddot{\theta} = \frac{5g}{7(R+r)}\theta$$

or

$$\theta(t) = \frac{\dot{\theta}(0)}{\omega}\sinh(\omega t)$$

where $\omega = \sqrt{\frac{5g}{7(R+r)}}$.

Príklad 4

a) $\dfrac{(2m|\epsilon_0|)^{1/2}}{\hbar}$

b) $K = e^{\alpha d}/\alpha^{1/2}$ $\qquad \left(\int_d^\infty |\psi_0|^2 dx = \frac{1}{2} \right)$

c) $\left(\dfrac{V_0}{\hbar}\right) e^{-2\alpha d}$

or $|\epsilon_0|/\hbar$

or $\hbar/ma^2$

The main point is the $e^{-2\alpha d}$

In our problem
$a \sim 1/\alpha$
$\langle KE \rangle \sim \langle P.E \rangle/2$
$\epsilon_0 \sim V_0/2$
so any dimension analysis "guess" gives some answe

d) $$\frac{\Psi_0(x) + \Psi_0(d-x)}{\sqrt{2}} \equiv \psi_{0s}$$

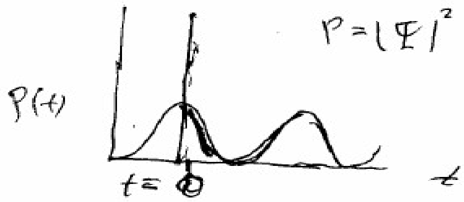
e) $\delta E = V_0 e^{-\alpha d}$

$$\underbrace{\int_0^\infty V(x)\,\psi_0(x)\,\psi_0(d-x)}\qquad \int_0^a \frac{V_0}{a} e^{-d\alpha}\,dx \quad dx$$

$$\Psi = \psi_{0s}\, e^{-i\delta E t/\hbar} + \psi_{Es}\, e^{\delta E t/\hbar}$$

$$\psi_{0s} = \frac{\psi_0(x) - \psi_0(d-x)}{\sqrt{2}}$$

f) $P = |\Psi|^2 \propto \psi_0^2 \cos^2 \delta E/\hbar + \psi_0^2(d-x) \sin^2 \delta E t/\hbar$



$$\underset{x \lesssim a}{P(t)} = \cos^2 t/\tau \qquad \frac{1}{\tau} = \frac{\delta E}{\hbar}$$

~~[illegible]~~ Ratio of times $\tau \sim e^{-\alpha d}$ for c) and $\hbar/\delta E$