METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto20 – Príklady 2

VZOROVÉ RIEŠENIA

Cvičenie 5.3.2020

Príklad 1

$\boxed{28.}$ Kľúčovým je uvedomenie si, že nás nezaujíma priebeh pohybu telieska, ale iba počiatočný a koncový stav a tiež fakt, že v gravitačnom poli sa celková energia telesa zachováva. Preto je zákon zachovania energie presne to, čo potrebujeme. Tak si ho pre našu sitáciu zapíšeme:

$$-\kappa\frac{M_1 m}{R_1} - \kappa\frac{M_2 m}{2R_1 + R_2} = \frac{1}{2}mv^2 - \kappa\frac{M_2 m}{R_2} - \kappa\frac{M_1 m}{R_1 + 2R2}$$

Odtiaľto už ľahko dostaneme hľadanú rýchlosť telieska:

$$v = 2\sqrt{\kappa\left(M_2\frac{R_1}{R_2(2R_1 + R_2)} - M_1\frac{R_2}{R_1(R_1 + 2R_2)}\right)}$$

Príklad 2

Príklad 3

$E = 0 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy$

$y = -ax^n \;\Rightarrow\; \dot{y} = -nax^{n-1}\dot{x}$

$\Rightarrow\; \dot{x}^2 = \frac{2gax^n}{1+n^2a^2x^{2(n-1)}}$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$2\cancel{\dot{x}}\,\ddot{x} = \frac{\partial}{\partial t}\left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}}\right) = \cancel{\dot{x}}\,\frac{\partial}{\partial x}\left(\frac{2gax^n}{1+n^2a^2x^{(2n-2)}}\right) = 0$

$\Rightarrow\; \frac{nx^{n-1}}{1+n^2a^2x^{2(n-1)}} - \frac{2(n-1)n^2a^2x^{3n-3}}{(1+n^2a^2x^{2n-2})^2} = 0$

$nx^{n-1} + n^3a^2x^{3n-3} - 2(n-1)n^2a^2x^{3n-3} = 0$

$\Rightarrow\; a^2x^{2n-2} = \frac{1}{n(n-2)}$. Real finite solution only for $n > 2$.

Príklad 4

(a) upper and lower branches are equivalent, so voltage across central capacitor is zero - throw it away.

$$C_a = 2\left(\frac{1}{C} + \frac{1}{2C}\right)^{-1} = \frac{4}{3}C$$

(b) From symmetry, voltage drops on outside C-capacitors are the same and we'll denote them U_1, and similarly for $2C$ capacitors, U_2, with condition $U_1 + U_2 = V$ - overall voltage drop. Voltage drop across middle one is $U_2 - U_1$. Write system of equations for charges on the capacitors, $q_1 = CU_1$, $q_2 = 2CU_2$ and $q_{center} = q_1 - q_2 = C(U_2 - U_1)$ and solve this system for $U_{1,2}$ and $q_{1,2}$. The overall capacitance is

$$C_b = \frac{q_1 + q_2}{V} = \frac{7}{5}C$$

(c) make a vertical cut after first vertical capacitor - resulting infinite chain is the same as the original

$$\frac{1}{C_c} = \frac{1}{C} + \frac{1}{C + C_c} \qquad \Rightarrow \qquad C_c = \frac{\sqrt{5} - 1}{2}C$$

(d) the points with the same voltages can be connected - this results in equivalent scheme: (3 parallel)-(6 parallel)-(3 parallel)

$$\frac{1}{C_d} = \frac{1}{3C} + \frac{1}{6C} + \frac{1}{3C} = \frac{5}{6C} \qquad \Rightarrow \qquad C_d = \frac{6}{5}C$$

Príklad 5

4.

$\overleftarrow{\frac{GMm}{(a-r)^2}}$ $\qquad$ $\overleftarrow{\frac{GMm}{(a+r)^2}}$ $\qquad$ FROM M

$\overrightarrow{\frac{Gm^2}{(2r)^2}}$ $\qquad$ $\overleftarrow{\frac{Gm^2}{(2r)^2}}$ $\qquad$ FROM EACHOTHER

FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE LESS THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{G\cancel{Mm}\,m^2}{4r^2}$$

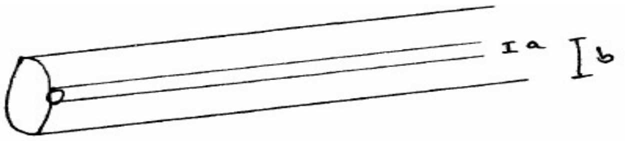
$$\frac{M}{(a^2-r^2)^2}(a+r)^2-\frac{M}{(a^2-r^2)^2}(a-r)^2<\frac{m}{2r^2}$$

$$4ar\,\frac{M}{a^4}<\frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3}<m=\frac{4}{3}\pi r^3\rho$$

$$\frac{6}{\pi}\frac{M}{a^3}<\rho$$

Príklad 6



magnetic fields using Ampère's law

$r>b \qquad \vec{B}=0$

$a<r<b \qquad \oint \vec{B}\,d\vec{l}=\mu_0 I \Rightarrow 2\pi s B=\mu_0 I$

$\Rightarrow \vec{B}=\frac{\mu_0 I}{2\pi s}\hat{\phi}$

$r<a \qquad 2\pi s B=\mu_0 I\frac{s^2}{a^2} \Rightarrow \vec{B}=\frac{\mu_0}{2\pi}\frac{Is}{a^2}\hat{\phi}$

$$W = \frac{1}{2\mu_0} \iiint B^2 \, dV$$, so per unit length

$$\frac{W}{\ell} = \frac{\mu_0}{2} \frac{I^2}{(2\pi)^2} 2\pi \left\{ \int_0^a \frac{s^2}{a^4} s \, ds + \int_a^b \frac{1}{s^2} \, s \, ds \right\}$$

$$= \frac{\mu_0}{4\pi} I^2 \left\{ \left[\frac{1}{4} \frac{s^4}{a^4} \right]_0^a + \left[\ln s \right]_a^b \right\}$$

$$= \frac{\mu_0}{16\pi} I^2 + \frac{\mu_0}{4\pi} \ln \frac{b}{a} I^2$$

Now use $W = \frac{1}{2} L I^2$, so $$\frac{L}{\ell} = \underbrace{\frac{\mu_0}{8\pi}}_{\text{vanishes for cylindrical shell}} + \frac{\mu_0}{2\pi} \ln b/a$$