METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima21 – Príklady 4

VZOROVÉ RIEŠENIA

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Príklad 1

$$L = \frac{m}{2}\left\{\dot{x}^2 + \left(\frac{l}{2}\right)^2 \cos^2\theta\,\dot{\theta}^2\right\} + \frac{ml^2}{24}\dot{\theta}^2 - \frac{mg}{2} l \sin\theta$$

$$\dot{x} = \text{const.} = 0 \quad , \quad x = 0$$

$$H = \left(\frac{m}{2}\frac{l^2}{4}\cos^2\theta + \frac{ml^2}{24}\right)\dot{\theta}^2 + \frac{mg}{2} l \sin\theta = \frac{mg}{2} l \sin\theta_0$$

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{(g/l)\,\frac{1}{2}\,(\sin\theta_0 - \sin\theta)}{\frac{1}{8}\cos^2\theta + \frac{1}{24}}$$

$$\sqrt{g/l}\; t = \int_0^{\theta_0} \sqrt{\frac{\frac{1}{4}\cos^2\theta + \frac{1}{12}}{\sin\theta_0 - \sin\theta}}\, d\theta$$

$$\Delta X_{end} = \frac{l}{2} - \frac{l}{2}\cos\theta_0$$

Príklad 2

(a) The Maxwell-Ampère law gives

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} .$$

With $\boldsymbol{j} = \sigma\boldsymbol{E}$, and with $\sigma \gg \omega$, we may drop the second term on the RHS. Taking the time derivative and invoking Faraday's law then gives

$$\begin{aligned} \frac{\partial}{\partial t}\nabla \times \boldsymbol{E} &\approx \frac{4\pi\sigma}{c}\frac{\partial \boldsymbol{E}}{\partial t} \\ &= -c\nabla \times \nabla \times \boldsymbol{E} \\ &= c\nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2\boldsymbol{E} , \end{aligned}$$

whence Gauss's law results in

$$\nabla^2\boldsymbol{E} = \frac{4\pi\sigma}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} .$$

Thus, we have

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{4\pi i\sigma\omega}{c^2} E_x = 0 .$$

The solution is of the form

$$E_x(z,t) = A\,e^{i(kz-\omega t)} + B\,e^{-i(kz+\omega t)}$$

where

$$k^2 = \frac{4\pi i\sigma\omega}{c^2} \quad \Longrightarrow \quad k = (1+i)\,\frac{\sqrt{2\pi\sigma\omega}}{c}$$

Since $\mathrm{Im}(k) > 0$, we must set $B = 0$ to have a valid solution at $z \to \infty$, in which case $A = E_0$. The penetration depth of the electric field is then

$$\ell = \frac{1}{\mathrm{Re}(k)} = \frac{c}{\sqrt{2\pi\sigma\omega}} .$$

(b) The power dissipated per unit area is

$$\frac{P}{A} = \tfrac{1}{2}\sigma \int_0^\infty dz \left|E(z,t)\right|^2$$

$$= \frac{\sigma E_0^2}{2\,\mathrm{Re}(k)} = \sqrt{\frac{\sigma c^2}{8\pi\omega}}\, E_0^2 \ .$$

Príklad 3

Answer:

$$C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT}$$

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don't have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by x balances pressure

$$pS = kx \qquad\qquad pV = pSx = kx^2 = \nu RT$$

and using this connection between x and T we get

$$C = \frac{3}{2}\nu R + kx\frac{dx}{dT} = \frac{3}{2}\nu R + kx\frac{1}{2kx}\nu R = 2\nu R \qquad\qquad \boxed{C = 2\frac{p_0 V_0}{T_0}}$$