METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima21 – Príklady 5

Cvičenie 16.11.2021

Príklad 1

a. A spherical region of space of radius R has a uniform charge density and total charge $+Q$. An electron of charge $-e$ is free to move inside or outside the sphere, under the influence of the charge density alone. For this first part ignore radiation effects.

  i. Consider a circular orbit for the electron where $r < R$. Determine the period of the orbit T in terms of any or all of r, R, Q, e, and any necessary fundamental constants.

  ii. Consider a circular orbit for the electron where $r > R$. Determine the period of the orbit T in terms of any or all of r, R, Q, e, and any necessary fundamental constants.

  iii. Assume the electron starts at rest at $r = 2R$. Determine the speed of the electron when it passes through the center in terms of any or all of R, Q, e, and any necessary fundamental constants.

b. Accelerating charges radiate. The total power P radiated by charge q with acceleration a is given by

$$P = C\xi a^n$$

where C is a dimensionless numerical constant (which is equal to $1/6\pi$), ξ is a physical constant that is a function only of the charge q, the speed of light c, and the permittivity of free space ϵ_0, and n is a dimensionless constant. Determine ξ and n.

c. Consider the electron in the first part, except now take into account radiation. Assume that the orbit remains circular and the orbital radius r changes by an amount $|\Delta r| \ll r$.

  i. Consider a circular orbit for the electron where $r < R$. Determine the change in the orbital radius Δr during one orbit in terms of any or all of r, R, Q, e, and any necessary fundamental constants. Be very specific about the sign of Δr.

  ii. Consider a circular orbit for the electron where $r > R$. Determine the change in the orbital radius Δr during one orbit in terms of any or all of r, R, Q, e, and any necessary fundamental constants. Be very specific about the sign of Δr.

Príklad 2

PROBLEM: A point charge $-2q$ is at the origin, $\mathbf{r} = 0$, and two point charges, each $+q$, are at $\mathbf{r} = \pm a\hat{z}$. Consider the limit $a \to 0$, with $Q = qa^2$ held fixed.

(a) Find the scalar potential $\phi(\mathbf{r})$ in spherical coordinates.

(b) This system of charges is now placed inside a grounded, conducting spherical shell, of radius b (with $b \gg a$). Now find the scalar potential $\phi(\mathbf{r})$ everywhere, both inside and outside of the shell (again, in spherical coordinates).

Príklad 3

PROBLEM: A particle of mass m is in the ground state of a harmonic oscillator with spring constant $k = m\omega^2$. At $t = 0$, the spring constant changes suddenly to $k' = \lambda^2 m\omega^2$, where λ is a constant. Find the probability that the oscillator remains in its ground state.

PROBLEM: Compute $\langle\psi_0|x^4|\psi_0\rangle$ in the ground state of the one-dimensional harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \tfrac{1}{2}m\omega^2 x^2 \ .$$