METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto22 – Príklady 3

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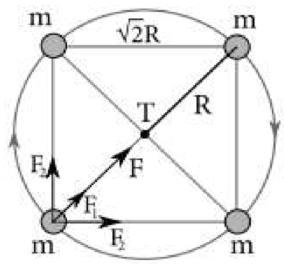
Príklad 1

Planéty na seba navzájom pôsobia gravitačnými silami. Situácia je zjavne úplne stredovo súmerná, takže môžeme spočítať, aká sila pôsobí na ľubovoľnú z nich.



Protiľahlá planéta sa nachádza vo vzdialenosti $2R$, preto pôsobí silou veľkosti $F_1 = \frac{Gm^2}{4R^2}$. Priľahlé planéty sa nachádzajú vo vzdialenosti $\sqrt{2}R$, preto budú pôsobiť silami veľkosti $F_2 = \frac{Gm^2}{2R^2}$. Zo symetrie úlohy je zrejmé, že výslednica síl od dvoch priľahlých planét bude smerovať do stredu, preto si ich rozložme do dostredného smeru a smeru naň kolmého. Kolmé zložky sa vybijú a prežijú iba dostredné zložky $\frac{1}{\sqrt{2}}F_2$.

Výsledná sila pôsobiaca na planétu teda bude

$$F = F_1 + 2 \cdot \frac{1}{\sqrt{2}}F_2 = \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)\frac{Gm^2}{R^2}.$$

Táto sila spôsobuje pohyb po kružnici, čiže je dostredivou silou $F = m\omega^2 R$. Z rovnosti dostredivej a výslednej gravitačnej sily dostávame $\omega = \sqrt{\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)\frac{Gm}{R^3}}$. Z definície uhlovej rýchlosti $\omega = \frac{2\pi}{T}$ dopočítame periódu obehu planét

$$T = \frac{4\pi}{\sqrt{2\sqrt{2}+1}}\sqrt{\frac{R^3}{Gm}}.$$

Príklad 2

Príklad 3

We need to find force acting on liquid in one quarter of the pipe. For a small segment of the pipe,

$$dF = \frac{dm\, v^2}{R} = \rho S d\phi\, v^2$$

and it is pointing towards the center, providing centripetal acceleration to the liquid. Projection of the total force on the horizontal axis is $F_x = \int_0^{\pi/2} dF \cos\phi$, and the answer is $T = F_x$,

$$\boxed{T = \rho S v^2}$$

This we can get from more sophisticated hydrodynamics as well, by considering momentum flux equation

$$\frac{\partial}{\partial t}\rho v_i = -\nabla_j \Pi_{ij} + f_i$$

where $\Pi_{ij} = \delta_{ij}p + \rho v_i v_j$ is momentum density tensor, and f_i is external force density. Considering left quarter of the pipe, for steady flow there is no momentum change in this volume, and thus we must have for the external force acting on the volume

$$F_i = \int dV\, f_i = \int dV\, \nabla_j \Pi_{ij} = \oint dS_j \Pi_{ij} = \oint \rho v_i (\mathbf{v} d\mathbf{S})$$

where we used Gauss theorem and the fact that the pressure is uniform. The surface integral is not zero only through opening parts of the pipe, since on the sides $\mathbf{v} \perp d\mathbf{S}$. For x-component of the force we integrate only over the vertical cross-section at the top of the semi-circle, to immediately get

$$T = F_x = \rho v(vS) = \rho S v^2$$

Príklad 4

ANGULAR MOMENTUM CONSERVATION:

INITIALLY $L = |\vec{r} \times \vec{p}| = mvr|\sin\theta| = mvb$

AT PERIGEE $L = mv_p r_p$

$\therefore\ v_p = vb/r_p$

ENERGY CONSERVATION: $KE + \frac{GMm}{r_p} \simeq 0$

$\therefore\ \frac{1}{2}mv_p^2 = \frac{GMm}{r_p}$

SUBSTITUTE FOR v_p: $\frac{v^2 b^2}{2r_p^2} = \frac{GM}{r_p}$

HENCE: $r_p = \frac{v^2 b^2}{2GM}$

Príklad 5

One can express the net radiative transfer, I_{net}, in terms of the left and right portions of the system.

$$I_R = \epsilon_L \sigma T_L^4 + (1-\epsilon_L) I_L$$

$$I_L = \epsilon_R \sigma T_R^4 + (1-\epsilon_R) I_R$$

$$I_{\text{net}} = I_R - I_L$$

By looking at the quantity $\epsilon_R I_R - \epsilon_L I_L$ and isolating $I_R - I_L$, one finds...

$$\boxed{I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{\epsilon_R + \epsilon_L - \epsilon_L \epsilon_R}}$$

For the infinite series approach, consider the contribution from each sequence. That is.

$$I_R' = \epsilon_R \epsilon_L \sigma T_L^4 (1 + (1-\epsilon_R)(1-\epsilon_L) + ((1-\epsilon_R)(1-\epsilon_L))^2 + ...)$$

and the same for I_L'. The infinite series term is just the geometric series with a value $(1-(1-\epsilon_R)(1-\epsilon_L))^{-1}$. Therefore, the net effect is

$$I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{1 - (1-\epsilon_R)(1-\epsilon_L)}$$

Which is the same as above.

Príklad 6

The linearity of Maxwell's equations allows us to find the magnetic field as a sum of two magnetic fields produced by two currents: A current with density

$$j_b = \frac{I}{\pi(b^2-a^2)} \tag{1}$$

carried by the cylinder of radius b and a current with density

$$j_a = -j_b \tag{2}$$

carried by a cylinder of radius a. The sum of these two currents gives the current distribution in the considered structure. From Ampere's circuital law $\oint \mathbf{H} \cdot d\mathbf{l} = \frac{4\pi}{c} \int \mathbf{j} \cdot d\mathbf{S}$ one finds that the current carried by the cylinder of radius b produces a magnetic field at the center of the hole

$$H_b = \frac{2Id}{c(b^2-a^2)}, \tag{3}$$

while the current carried by the cylinder of radius a produces no magnetic field at the center of the hole, $H_a = 0$. Therefore, the magnetic field at the center of the hole is

$$H = \frac{2Id}{c(b^2-a^2)}. \tag{4}$$