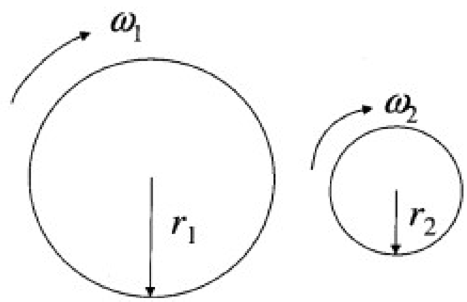
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto22 – Príklady 4

Cvičenie 31.3.2022

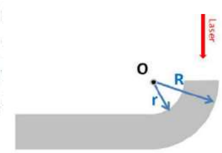
Príklad 1

Two uniform cylinders spin independently about their axes (the axes are parallel to each other). The first has radius r_1 and mass m_1, the other has radius r_2 and mass m_2. Initially they rotate in the same sense of rotation with angular speeds ω_1 and ω_2 respectively. They are then brought together so that they touch. After the steady state is achieved, what is the final angular velocity of cylinder 1, ω_1'?



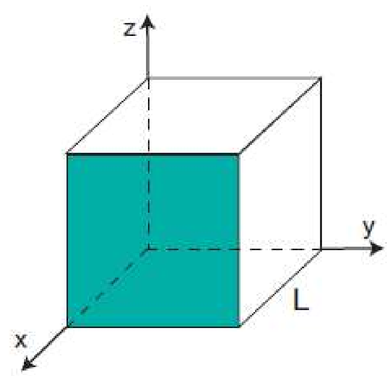
Príklad 2

Consider a thick sheet of glass (index of refraction n) that is bent at one end so the cross section is as shown in the figure. The bend is perfectly cylindrical and the inner radius is r and the outer radius is R. A laser is pointing vertically down and enters the glass through the top side. What is the constraint that ensures that the laser light exists through the left (vertical) face and nowhere else?



Príklad 3

Consider a three-dimensional box with sides of length L, as shown



It contains an ideal gas of non-interacting spin-less particles each with kinetic energy

$$\varepsilon = \frac{m}{2}\vec{v}^2$$

The temperature of the gas is T, and the particles are uniformly distributed throughout the box.

a) What is the normalized velocity distribution of the gas?

b) We now open the front side of the box (the shaded side facing the $+x$-direction, as shown in the figure) for a given time Δt. Using the result from (a), compute the number of particles that escape from the box in time Δt. To this end, consider these two steps: (i) Divide the box into slices of width dx and compute first the number of particles in a given slice at a distance x from the opening that have escape through the opening in time Δt. (ii) In order to find the total number of escaped particles, integrate the result you obtained in (i).

c) How does the total number of escaped particles depend on Δt in the limit $\Delta t \to 0$?

Príklad 4

(a) Consider two Hermitian operators A and B on a Hilbert space which satisfy $(A+B)^2 = 2\,AB$. Show that $A = B = \hat{0}$.

(b) Consider a quantum system with two-dimensional Hilbert space $\mathcal{H}$, and assume that the system is in the state $|\psi\rangle$. Find two Hermitian operators A and B on $\mathcal{H}$ such that if a measurement of A is made on the system, followed immediately by a measurement of B, the state of the system immediately after the second measurement has a non-zero probability of being orthogonal to $|\psi\rangle$. (Hint: Consider an orthonormal basis for $\mathcal{H}$ containing $|\psi\rangle$, and express A and B in this basis.)