METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 3 leto22 – Príklady 6

Cvičenie 27.4.2021

Príklad 1

Problem 1. [Classical Mechanics] Brad Majors and Janet Weiss go bowling. Ever observant, Brad notes that his bowling ball initially skids down the alley, and only slowly begins to roll.

a) Briefly, and in words, why does a ball that initially skids down the alley eventually start to roll down the alley?
b) The bowling ball has radius R. What mathematical constraint holds when the ball begins to roll without slipping?
c) The ball is released with an initial translational speed of v_0 and an initial rotational speed of zero. The coefficient of kinetic friction between the ball and the alley is μ and the mass of the ball is M. Derive an expression for the time, denoted T, between the release of the ball and the onset of rolling without slipping [minor help: $I_{\text{sphere}} = (2/5)MR^2$].

Príklad 2

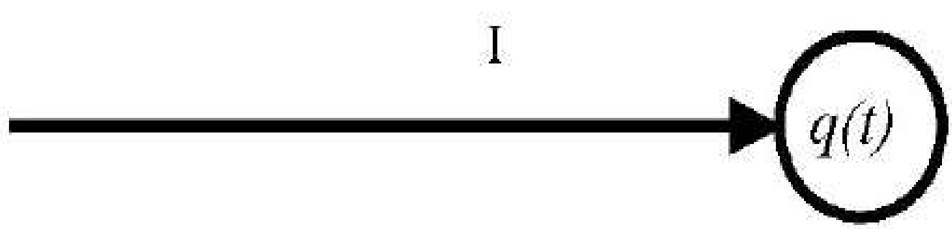
PROBLEM: This problem deals with displacement current.

(a) Consider the displacement current $\boldsymbol{j}_\text{d} = \frac{1}{4\pi}\frac{\partial \boldsymbol{E}}{\partial t}$ of an electromagnetic field, and using Maxwell's equations show that the sum $\boldsymbol{J} = \boldsymbol{j} + \boldsymbol{j}_\text{d}$ is divergenceless: $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. (Here, $\boldsymbol{j}$ is the current of charges.)

(b) A conducting sphere of radius a is being charged through a straight wire, carrying current I, so that the charge on the sphere q obeys $\dot{q} = I$. Assuming a symmetric distribution of charge over the sphere's surface, find the electric field outside the sphere. Determine also the displacement current, and verify the conservation law $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$.

(c) Using the Ampére-Maxwell law in the form $\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J}$, and taking advantage of the cylindrical symmetry in this problem, find the magnetic field everywhere in space.

(d) By appropriately limiting your result from (c), verify that close to the wire, the answer has the familiar form for an infinite straight wire.



Príklad 3

PROBLEM: The figure below illustrates a simple model of the surface of a two-dimensional 'solid' confined to a square lattice. The two ends of the surface are N lattice sites apart, with $N \gg 1$. The surface energy is proportional to the surface length, with energy $\varepsilon > 0$ per lattice length. The surface height can change by at most one lattice length at a time. (Overlaps are forbidden, so that outward-pointing surface normals never point downward.) Thus, the surface can be modeled by a Hamiltonian

$$H = \varepsilon \sum_{i=1}^{N} (1 + \sigma_i^2) \ ,$$

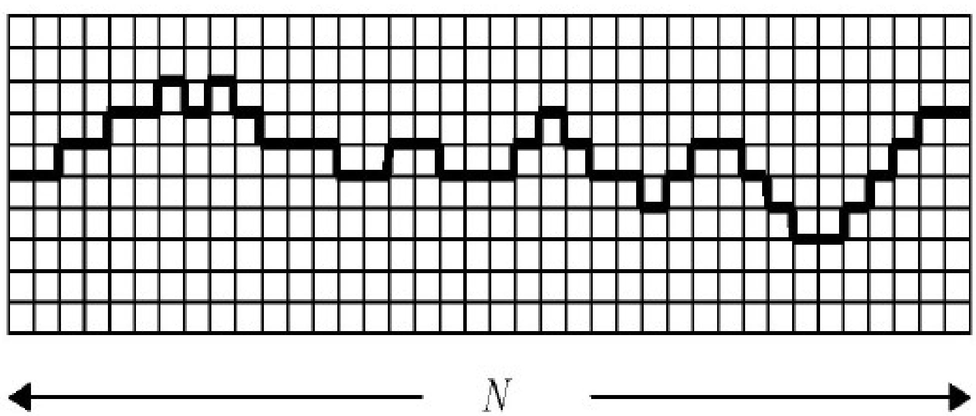
where $\sigma_i = +1$, 0, or -1 depending on whether the i^{th} 'column' contains a step up, no step, or a step down for the surface.

(a) Explain why this is Hamiltonian properly reflects the surface energy described above.

(b) Find the partition function $Z(T)$ for the surface.

(c) Find the free energy $F(T)$ for the surface, and sketch its temperature dependence. Physically interpret your result in the limits $k_{\text{B}}T \ll \varepsilon$ and $k_{\text{B}}T \gg \varepsilon$.

(d) Find the total length of the surface as a function of temperature, and sketch its temperature dependence.



Príklad 4

A two level system has as its Hamiltonian

$$\mathcal{H} = \begin{pmatrix} 0 & ig \\ -ig & \Delta \end{pmatrix}$$

in some basis. At time zero, the quantity D, described by

$$D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

is measured and found to have the value zero.

1. What is the probability that a measurement of D at a later time t will yield the value one?

2. If a measuring apparatus monitors the value of D continuously, what is the probability that its value will be one at the later time t?