METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima21 – Príklady 3

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Príklad 1

$$L = \frac{m}{2}\left\{\dot{x}^2 + \left(\frac{l}{2}\right)^2 \cos^2\theta\,\dot{\theta}^2\right\} + \frac{ml^2}{24}\dot{\theta}^2 - \frac{mgl}{2}\sin\theta$$

$$\dot{x} = \text{const.} = 0 \quad , \quad x = 0$$

$$H = \left(\frac{m}{2}\frac{l^2}{4}\cos^2\theta + \frac{ml^2}{24}\right)\dot{\theta}^2 + \frac{mgl}{2}\sin\theta = \frac{mgl}{2}\sin\theta_0$$

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{(g/l)\,\frac{1}{2}\,(\sin\theta_0 - \sin\theta)}{\frac{1}{8}\cos^2\theta + \frac{1}{24}}$$

$$\sqrt{g/l}\;t = \int_0^{\theta_0} \sqrt{\frac{\frac{1}{4}\cos^2\theta + \frac{1}{12}}{\sin\theta_0 - \sin\theta}}\,d\theta$$

$$\Delta X_{end} = \frac{l}{2} - \frac{l}{2}\cos\theta_0$$

Príklad 2

Solution

a) $r > R \quad \phi_o(r\theta) = -E_0 r\cos\theta + \sum_0^\infty a_n r^{-(n+1)} P_n(\cos\theta)$

$S \leq r \leq R \quad \phi_i(r\theta) = \sum_0^\infty \left(b_n r^n + c_n r^{-(n+1)}\right) P_n$

You can see only terms 0,1 are required to match boundary conditions.

$\underline{\phi_i(S\,\theta) = 0} \Rightarrow \quad b_0 + \frac{c_0}{S} = 0 \quad c_0 = -b_0 S \quad P_0$

$b_1 S + \frac{c_1}{S^2} = 0 \quad c_1 = -b_1 S^3 \quad P_1$

$\phi_o(R) = \phi_i(R)$ $\qquad b_o(1 - S/R) = a_o/R \qquad P_o$

① $\boxed{b_1 R\left(1 - \frac{S^3}{R^3}\right) = -E_o R + \frac{a_1}{R^2}} \qquad P_1$

$-\left.\frac{\partial \phi_o}{\partial r}\right|_R = -\varepsilon_r \left.\frac{\partial \phi_i}{\partial r}\right|_R$ (D-field normal component continuous)

$-S/R^2\, b_o \varepsilon_r = a_o/R^2 \qquad P_o$

② $\boxed{-b_1\varepsilon_r - 2\frac{S^3}{R^3} b_1 \varepsilon_r = E_o + \frac{2a_1}{R^3}} \qquad P_1$

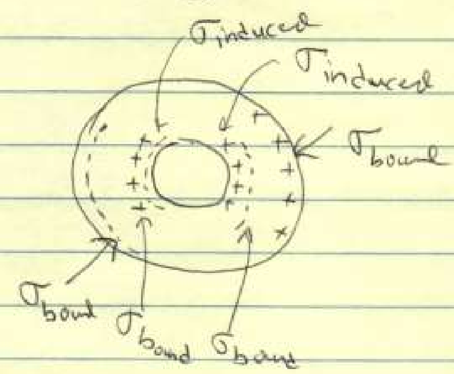
$\Rightarrow a_o = b_o = 0$

From ① + ② $\quad b_1 = \dfrac{-E_o}{1 - \frac{S^3}{R^3} - \frac{1}{3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}$

Ans 2 $\quad \phi_o(r\theta) = -E_o r\cos\theta \left\{1 + \dfrac{\frac{R^3}{3r^3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}{1 - \frac{S^3}{R^3} - \frac{1}{3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}\right\}$

A quick check shows that for $S \to 0$ this approaches the dielectric sphere in uniform $\vec{E}$ and for $R \to S$, $\varepsilon_r \to 1$ it approaches the conducting shell in uniform $\vec{E}$.

b)



All charges decreasing towards $\theta = \pi/2$

Príklad 3

Solution. The boat ends up where it started! The mass of the man does not matter, nor do the length and the mass of the boat, nor does the magnitude of the drag coefficient k. The problem requires *no* data!

A description of the motion. (A precise solution is given in the next paragraph.) When the person starts walking right, the boat starts moving left (Figure 14.5). Hence drag force points right, and by Newton's second law the center of mass

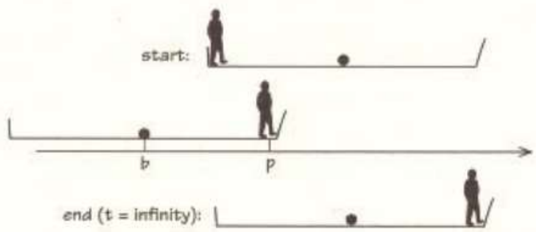


Figure 14.5. The boat eventually approaches its starting position.

of the man+boat accelerates right.[6] Having acquired motion to the right, the center of mass of man + boat continues by inertia even after the man sits down. Eventually, all slows down due to drag. To recap: the boat started moving left, activating the drag, which caused the center of mass of the whole system to move right—the motion that persisted once the man sat down. Remarkably, the boat will approach its initial position as time goes on. Why this strange coincidence happens is still unclear from the argument just given, but it is explained in the next paragraph.

The justification of the remarkable answer is quite simple, but it requires a little calculus. Let $b = b(t)$ denote the position at time t of the boat's center of mass (all is measured in a reference frame of the shore), and, similarly, let $p = p(t)$ be the position of the person (treated as a point mass). The center of mass[7] of the boat–person system is the weighted average of the two positions: $C = C(t) = (mp + Mb)/(m + M)$. Newton's second law (stated on page 172), applied in the direction of the boat's motion, gives

$$(m + M)\ddot{C} = -k\dot{b};$$

here each dot denotes the time derivative. Substituting the expression for C we obtain

$$m\ddot{p} + M\ddot{b} = -k\dot{b}. \qquad (14.1)$$

Let us integrate this relation from $t = 0$ to $t = \infty$. The Fundamental Theorem of Calculus[8] gives $\int_0^\infty \ddot{p}\, dt = \dot{p}(\infty) - \dot{p}(0)$. But $\dot{p}(0) = 0$ because all starts at rest, and

[6] This is a little like a cartoon dog running almost in place: his feet slide backward on the ground (like the boat sliding in water), while the dog's center of mass accelerates forward. Cartoon characters, however, routinely break Newton's laws.

[7] See page 169 for the definition.

[8] See page 184 for the details.