METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima20 – Príklady 5

VZOROVÉ RIEŠENIA

Cvičenie 1.12.2022

Príklad 1

4. Two blocks and two pulleys (Question from Dave, solution from Peter)

Most straightforward to use Lagrangian with constraint for fixed length of rope:

$L = \frac{1}{2}\left(m_1\dot{x}^2 + m_2\dot{y}^2\right) + m_1 g x$ where (referring to the diagram with the problem) x

$g(x, y) = 0 = x + 2y + d - l$

increases downward and y increases to the right. The rope has length l and d accounts for all the rope not taken up by x and y. Also from the constraint, we have $\ddot{x} = -2\ddot{y}$. Using the Euler-Lagrange equations with undetermined multiplier gives

$$\begin{matrix} m_1\ddot{x} - m_1 g + \lambda = 0 \\ m_2\ddot{y} + 2\lambda = 0 \end{matrix} \Rightarrow \begin{matrix} -2m_1\ddot{x} + 2m_1 g - 2\lambda = 0 \\ -\frac{m_2\ddot{x}}{2} + 2\lambda = 0 \end{matrix} \Rightarrow \ddot{x} = \frac{4m_1 g}{4m_1 + m_2}$$

Check limits: $m_1 >> m_2$, acceleration is g, : $m_1 << m_2$, acceleration goes to zero.

4. $\frac{GMm}{(a-r)^2} \leftarrow$ $\quad \frac{GMm}{(a+r)^2} \leftarrow$ FROM M

$\rightarrow \frac{Gm^2}{(2r)^2}$ $\quad \leftarrow \frac{Gm^2}{(2r)^2}$ FROM EACHOTHER

FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE <u>LESS</u> THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{G\cancel{Mm}\,m^2}{4r^2}$$

$$\frac{M}{(a^2-r^2)^2}(a+r)^2 - \frac{M}{(a^2-r^2)^2}(a-r)^2 < \frac{m}{2r^2}$$

$$4ar\,\frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\underline{\underline{\frac{6}{\pi}\frac{M}{a^3} < \rho}}$$

Príklad 2

3. Current carrying wire (Question and solution from Peter)

a) Take loop of radius r around wire and use Ampere's law: $\oint \vec{B}\cdot d\vec{l} = {}^{4\pi}\!/_{c}\, I \Rightarrow \vec{B} = B(r)\hat{\varphi} = \frac{2I}{rc}\hat{\varphi} \Rightarrow \vec{F} = q\frac{\vec{v}}{c}\times\vec{B} = \frac{2Ive}{rc^2}\hat{\rho}$ where $\hat{\rho}$ is radial direction in cylindrical coordinates.

b) If F' moves along wire with v, then particle is at rest in F' and the Lorentz force is zero. Transform force from F: $F_\perp = \frac{dp_\perp}{dt} = \frac{dp'_\perp}{\gamma dt' - \beta\gamma dx'} = \frac{dp'_\perp/dt'}{\gamma + \beta\gamma\, dx'/dt'} = \frac{F'_\perp}{\gamma} \Rightarrow F'_{\perp=} = \gamma\frac{2Ive}{rc^2}$. Can also do problem by transforming currents and charge densities.

Príklad 3

SOLUTION: For a non-interacting ideal gas,

$$E = -\frac{\partial}{\partial\beta} N \ln\zeta\,,$$

where ζ is the single-molecule partition function

$$\zeta = \sum_{n=0}^{\infty}(n+1)\exp(-\beta n\varepsilon)\,.$$

This partition function can be evaluated as follows ($x \equiv \beta\varepsilon$):

$$\zeta = -e^x\frac{d}{dx}\sum_{n=0}^{\infty}\exp\big(-(n+1)x\big) = -e^x\frac{d}{dx}\frac{e^{-x}}{1-e^{-x}} = [1-\exp(-\beta\varepsilon)]^{-2}.$$

Hence, the sought contribution to the energy is

$$E = \frac{2N\varepsilon}{\exp(\varepsilon/kT)-1}\,.$$

Alternatively, one can reproduce this result as follows. One can imagine that every molecule has two independent internal degrees of freedom of harmonic oscilator type, with energy spacing ε each. It is easy to see that this model gives the same spectrum and degeneracies if the energy is counted from the ground state. With this convention, the average energy of a single harmonic oscillator is $\varepsilon n_B(\varepsilon)$, where $n_B(\varepsilon)$ is the Bose-Einstein occupation number. Therefore, for the entire gas we get $E = 2N\varepsilon n_B(\varepsilon)$, in agreement with the first derivation.