METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima22 – Príklady 6

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Príklad 1

a. In order for the masses to collide, the total angular momentum of the system must be zero, which only occurs if $v_0 = 0$.

b. In this case, the masses undergo uniform circular motion with radius $\frac{l}{2}$ and speed v_0, so that

$$\frac{Gm^2}{l^2} = \frac{m{v_0}^2}{\frac{l}{2}}$$

$$\frac{Gm}{{v_0}^2 l} = 2$$

c. The masses follow closed orbits if they do not have enough energy to escape, *i.e.* if the total energy of the system is negative. The total energy of the system is

$$2 \cdot \frac{1}{2} m{v_0}^2 - \frac{Gm^2}{l}$$

so that the condition required is

$$m{v_0}^2 - \frac{Gm^2}{l} < 0$$

$$\frac{Gm}{{v_0}^2 l} > 1$$

d. Note that the masses will always move symmetrically about the center of mass. Thus, in order to be at minimum separation, their velocities must be perpendicular to the line joining them (and will be oppositely directed). Let the minimum separation be d, and let the speed of each mass at minimum separation be v.

$$L = 2mv\frac{d}{2} = mvd$$

The initial angular momentum is likewise $mv_0 l$, and so by conservation of angular momentum

$$mvd = mv_0 l$$

$$v = v_0 \frac{l}{d}$$

By conservation of energy

$$2 \cdot \frac{1}{2} m{v_0}^2 - \frac{Gm^2}{l} = 2 \cdot \frac{1}{2} mv^2 - \frac{Gm^2}{d}$$

$${v_0}^2 - \frac{Gm}{l} = v^2 - \frac{Gm}{d}$$

Combining these,

$${v_0}^2 - \frac{Gm}{l} = {v_0}^2 \frac{l^2}{d^2} - \frac{Gm}{d}$$

$$\left(1 - \frac{Gm}{{v_0}^2 l}\right)\left(\frac{d}{l}\right)^2 + \frac{Gm}{{v_0}^2 l}\left(\frac{d}{l}\right) - 1 = 0$$

$$\left(\frac{d}{l} - 1\right)\left(\left(1 - \frac{Gm}{{v_0}^2 l}\right)\frac{d}{l} + 1\right) = 0$$

so that

$$d = l \quad \text{or} \quad d = \frac{l}{\frac{Gm}{{v_0}^2 l} - 1}$$

The second root is only sensible if $\frac{Gm}{{v_0}^2 l} > 1$, and is only smaller than the first if $\frac{Gm}{{v_0}^2 l} > 2$. (Note that both of these results make sense in light of the previous ones.) Thus the minimum separation is l if $\frac{Gm}{{v_0}^2 l} \leq 2$ and $\frac{l}{\frac{Gm}{{v_0}^2 l} - 1}$ otherwise.

Answer:

$$C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT}$$

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don't have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by x balances pressure

$$pS = kx \qquad pV = pSx = kx^2 = \nu RT$$

and using this connection between x and T we get

$$C = \frac{3}{2}\nu R + kx\frac{dx}{dT} = \frac{3}{2}\nu R + kx\frac{1}{2kx}\nu R = 2\nu R \qquad \boxed{C = 2\frac{p_0 V_0}{T_0}}$$

Príklad 2

Let the surrounding temperature be T_0. The rate of energy loss of the black body before being surrounded by the spherical shell is

$$Q = 4\pi r^2 \sigma (T^4 - T_0^4)\ .$$

The energy loss per unit time by the black body after being surrounded by the shell is

$$Q' = 4\pi r^2 \sigma (T^4 - T_1^4),\ \text{where } T_1 \text{ is temperature of the shell .}$$

The energy loss per unit time by the shell is

$$Q'' = 4\pi R^2 \sigma (T_1^4 - T_0^4)\ .$$

Since $Q'' = Q'$, we obtain

$$T_1^4 = (r^2 T^4 + R^2 T_0^4)/(R^2 + r^2)\ .$$

Hence $Q'/Q = R^2/(R^2 + r^2)$, i.e., $a = 1$ and $b = 1$.

Príklad 3

The perturbation $\Delta h(\mathbf{r})$ of the water height at a given point $\mathbf{r}$ is related to the local perturbation of the potential energy ΔU per unit mass:

$$\Delta h(\mathbf{r}) = -\Delta U(\mathbf{r})/g\,.$$

Denote the Earth's mass by M, then the mass of the Moon is μM. Let us choose a reference frame with the origin at the center of the Earth and the z-axis directed towards the Moon. The position of the Moon is therefore $\mathbf{R} = (0, 0, R)$. The potential energy in question is given by

$$U(\mathbf{r}) = -\frac{G\mu M}{|\mathbf{r}-\mathbf{R}|} + za\,.$$

Here the first term is the gravitational potential created by the Moon. The second term, linear in z, has the following origin. In the Earth-Moon center-of-mass frame, the Earth falls onto the Moon with acceleration $a = G\mu M/R^2$. However, in our reference frame the Earth is stationary but there exists an apparent acceleration $-a$, which is equivalent to an effective gravitational field a directed away from the Moon.

Expanding U to the leading nontrivial order in $\mathbf{r}$, we get

$$U(\mathbf{r}) \approx -\tfrac{G\mu M}{R} + \tfrac{G\mu M}{2R^3}(r^2 - 3z^2)\,,$$

which has the expected dipolar form. Using $r = R_0$, $z = R_0 \cos\theta$, where θ is the polar angle, and also $g = GM/R_0^2$, we get

$$\Delta h(\theta) = \tfrac{\mu R_0^4}{2R^3}(3\cos^2\theta - 1)\,, \quad h_{\max} - h_{\min} = \tfrac{3\mu R_0^4}{2R^3} \approx 55\,\mathrm{cm}\,.$$