METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 2 zima23 – Príklady 2

VZOROVÉ RIEŠENIA

Cvičenie 12.10.2023

Príklad 1

(a) Let v_0 denote the asymptotic common speed.

In a given time interval Δt, the cluster collides with $v_0 \Delta t/d$ further beads, which increases its mass by $\Delta m = m v_0 \Delta t/d$ and its momentum by $\Delta p = v_0 \Delta m = m v_0^2 \Delta t/d$. According to Newton's law of motion,

$$F = \frac{\Delta p}{\Delta t} = \frac{m v_0^2}{d},$$

which yields $v_0 = \sqrt{Fd/m}$ for the ultimate speed in the case of inelastic collisions.

(b) In an elastic collision between two equal mass bodies with one of them initially at rest, their velocities are exchanged. The body initially moving with velocity v stops, while the second one moves away with velocity v.

The leftmost bead accelerates uniformly and reaches a speed of

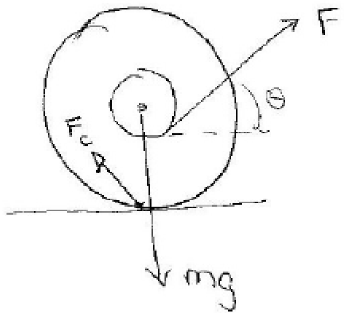
$$v_1 = \sqrt{\frac{2Fd}{m}} = \sqrt{2} v_0$$

before the first (elastic) collision takes place. It then transfers its speed to the second bead and stops, after which it starts accelerating again as a result of the external force. The second bead moves at a constant speed v_1, collides with the third bead and stops. The third and subsequent beads behave similarly, and a 'shock wave' propagates forward at speed v_1.

Meanwhile, the leftmost bead is again accelerated to speed v_1, collides with the second bead, which is now at rest, and the process is repeated, thus starting a new 'shock wave'. The speed of the leftmost bead varies uniformly from zero to v_1, its average value is $v_1/2 = v_0/\sqrt{2} = \sqrt{Fd/(2m)}$.

Príklad 2

2.



y: $\Sigma F_y = F_{cy} + F\sin\theta - mg = 0$

x: $\Sigma F_y = -F_{cx} + F\cos\theta = MR\frac{d\omega}{dt}$

τ about CM $\Sigma \vec{\tau} = +F_{cx}R - Fr = I_0 \frac{d\omega}{dt}$

$$F_{cx} = F\cos\theta - MR\frac{d\omega}{dt}$$

$$\left(F\cos\theta - MR\frac{d\omega}{dt}\right)R - Fr = I_0\frac{d\omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{FR\cos\theta - Fr}{I_0 + MR^2}$$

Forward motion occurs when $\frac{d\omega}{dt} > 0$ if

$FR\cos\theta > Fr$ $\quad$ $\cos\theta > \frac{r}{R}$

Backward motion occurs when $\cos\theta < \frac{r}{R}$

So. $\cos\theta_c = \frac{r}{R}$

Príklad 3

O separácii premenných sa dá dočítať čosi napríklad aj tu

https://phys.libretexts.org/Bookshelves/Electricity_and_Magnetism/Electromagnetics_and_Applications_(Staelin)/04%3A_Static_and_Quasistatic_Fields/4.05%3A_Laplace%E2%80%99s_equation_and_separation_of_variables .

Answer: In general $\nabla^2\phi = 0$ in the box gives

$$\phi(x,y,z) = \sum_{m_1,m_2} \sin\frac{m_1\pi x}{a}\sin\frac{m_2\pi y}{b}\left(A_{m_1m_2}\sinh K_{m_1m_2}z + B_{m_1m_2}\cosh K_{m_1m_2}z\right)$$

because $\phi\big|_{x=0,a} = \phi\big|_{y=0,b} = 0$. In the above

$$K^2_{m_1m_2} = \left(\frac{m_1\pi}{a}\right)^2 + \left(\frac{m_2\pi}{b}\right)^2 .$$

$$\phi\big|_{z=0} = V_1 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \Rightarrow B_{m_1m_2} = V_1\delta_{m_1 1}\delta_{m_2 2}$$

$$\phi\big|_{z=c} = V_2 \sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b} \Rightarrow \boxed{A_{22} = \frac{V_2}{\sinh K_{22}c}}$$

and $V_1\cosh K_{11}c + A_{11}\sinh K_{11}c = 0$

$$\boxed{A_{11} = -V_1\frac{\cosh K_{11}c}{\sinh K_{11}c}}$$

All other $A_{ij} = 0$

Príklad 4

(a) The Maxwell-Ampère law gives

$$\boldsymbol{\nabla}\times\boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t} .$$

With $\boldsymbol{j} = \sigma\boldsymbol{E}$, and with $\sigma \gg \omega$, we may drop the second term on the RHS. Taking the time derivative and invoking Faraday's law then gives

$$\begin{aligned}\frac{\partial}{\partial t}\boldsymbol{\nabla}\times\boldsymbol{E} &\approx \frac{4\pi\sigma}{c}\frac{\partial\boldsymbol{E}}{\partial t}\\ &= -c\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}\\ &= c\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{E}) - \nabla^2\boldsymbol{E} ,\end{aligned}$$

whence Gauss's law results in

$$\nabla^2\boldsymbol{E} = \frac{4\pi\sigma}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} .$$

Thus, we have

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{4\pi i\sigma\omega}{c^2}E_x = 0 .$$

The solution is of the form

$$E_x(z,t) = A\,e^{i(kz-\omega t)} + B\,e^{-i(kz+\omega t)}$$

where

$$k^2 = \frac{4\pi i\sigma\omega}{c^2} \quad\Longrightarrow\quad k = (1+i)\,\frac{\sqrt{2\pi\sigma\omega}}{c}$$

Since $\mathrm{Im}(k) > 0$, we must set $B = 0$ to have a valid solution at $z \to \infty$, in which case $A = E_0$. The penetration depth of the electric field is then

$$\ell = \frac{1}{\mathrm{Re}(k)} = \frac{c}{\sqrt{2\pi\sigma\omega}}\ .$$

(b) The power dissipated per unit area is

$$\begin{aligned}\frac{P}{A} &= \tfrac{1}{2}\sigma\int\limits_0^\infty\! dz\,\big|E(z,t)\big|^2\\ &= \frac{\sigma E_0^2}{2\,\mathrm{Re}(k)} = \sqrt{\frac{\sigma c^2}{8\pi\omega}}\,E_0^2\ .\end{aligned}$$