METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima22 – Príklady 4

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Príklad 1

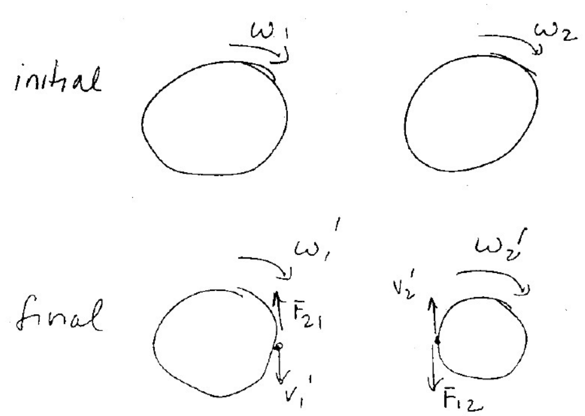
- Introduce the tension T in the rope and the force F which the mass M_h exerts to the right on the sphere. Use X, Y and x for the laboratory coordinates of M_h, M_v and m respectively.
- Write down equations for the acceleration of the cm of each mass:

$$\begin{aligned} M_h\ddot{X} &= T - F \\ m\ddot{a} &= F \\ M_v\ddot{Y} &= T - M_v g \end{aligned}$$

- Let θ represent the angular orientation of the sphere (increasing with clockwise motion) and write the equation for the angular acceleration of the sphere and the relation between $\ddot{\theta}$, $\ddot{a}$ and $\ddot{X}$:

$$\begin{aligned} \frac{2}{5}mR^2\ddot{\theta} &= -FR \\ \ddot{a} &= \ddot{X} + R\ddot{\theta} \end{aligned}$$

- These five equations can then be solved for T, F, $\ddot{\theta}$, $\ddot{X}$, $\ddot{Y}$,



Since the tangential final speeds must be the same (but opposite directions)

$$\omega_1' r_1 = -\omega_2' r_2 \quad (5)$$

$$\Rightarrow \quad \omega_2' = -\frac{r_1}{r_2}\omega_1'$$

Since the integral of the torque equals the change in angular momentum

$$I_1(\omega_1' - \omega_1) = -\int |r_1||F_{21}|dt \quad (5)$$

and $I_2(\omega_2' - \omega_2) = -\int |r_2||F_{12}|dt$ but $|F_{21}| = |F_{12}|$ from Newtons 3rd law

$$\therefore \quad \frac{I_1}{r_1}(\omega_1' - \omega_1) = \frac{I_2}{r_2}(\omega_2' - \omega_2)$$

Subst in for $\omega_2' = -\frac{r_1}{r_2}\omega_1'$

$$\therefore \frac{I_1}{r_1}\omega_1' + \frac{I_2}{r_2}\cdot\frac{r_1}{r_2}\omega_1' = \frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2$$

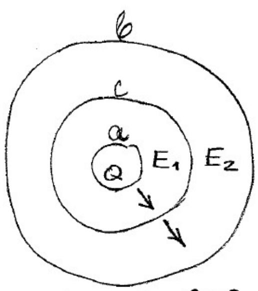
$$\omega_1' = \frac{\frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2}{\frac{I_1}{r_1} + I_2\frac{r_1}{r_2}} = \frac{\frac{L_1}{r_1} - \frac{L_2}{r_2}}{\frac{I_1}{r_1} + \frac{I_2 r_1}{r_2}}$$

with $I = \frac{1}{2}mr^2$ and some algebra (2)

$$\boxed{\omega_1' = \frac{m_1 r_1 \omega_1 - m_2 r_2 \omega_2}{(m_1+m_2) r_1}} \quad (3)$$

Príklad 2

3.



Assuming Q>0

(a) Gauss law:

$$D = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2}$$

$$E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2}$$

(b)

$$\phi_a - \phi_c = \int_a^c dr\, E_1 = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right)$$

$$\phi_b - \phi_c = \int_c^b dr\, E_2 = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

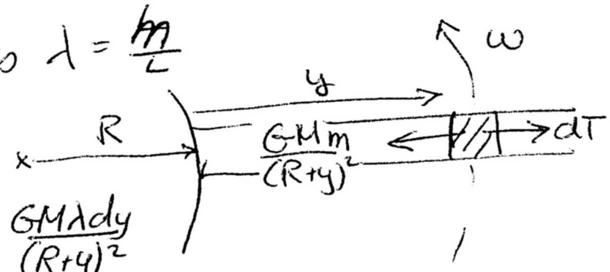
$$\phi_a - \phi_b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]$$

$$C = \frac{Q}{\phi_a - \phi_b} = 4\pi\varepsilon_0\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]^{-1} \quad \checkmark$$

(c) Gauss law:

$$\sigma = \varepsilon_0(E_2 - E_1)\Big|_{r=c} = \frac{Q}{4\pi\varepsilon_0 c^2}\left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1}\right) \quad \checkmark$$

Príklad 3



A. The density is $\lambda = \frac{m}{L}$

$$\frac{GMm}{(R+y)^2} + dT = m\omega^2(R+y)$$

$$\therefore dT = \lambda\, dy\, \omega^2(R+y) - \frac{GM\lambda\, dy}{(R+y)^2}$$

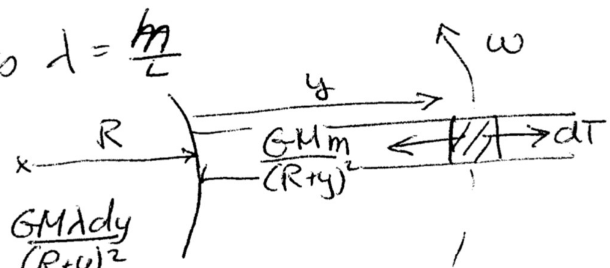
$$T(y) = \frac{GM\lambda}{(R+y)} + \frac{\lambda\omega^2}{2}(R+y)^2 \Big|_y^L$$

$$\therefore T(y) = \frac{GM\lambda}{R+L} - \frac{GM\lambda}{R+y} + \frac{\lambda\omega^2(R+L)^2}{2} - \frac{\lambda\omega^2(R+y)^2}{2}$$

$$= GM\lambda\left(\frac{1}{R+L} - \frac{1}{R+y}\right) + \frac{\lambda\omega^2}{2}\left[R^2+2LR+L^2-R^2-2Ry-y^2\right]$$

$$= GM\lambda\left(\frac{y-L}{(R+L)(R+y)}\right) + \frac{\lambda\omega^2}{2}\left(2R(L-y)+L^2-y^2\right)$$

$$\boxed{T(y) = (L-y)\lambda\left[\frac{-GM}{(R+L)(R+y)} + \frac{\omega^2}{2}(2R+L+y)\right]}$$



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Príklad 4

Príklad vzorák nemá, snáď to nie je problém ☺ Keby bolo treba, dohodnite si kozultáciu.