METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima22 – Príklady 5

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Príklad 1

4. Two blocks and two pulleys (Question from Dave, solution from Peter)

Most straightforward to use Lagrangian with constraint for fixed length of rope:

$L = \frac{1}{2}\left(m_1\dot{x}^2 + m_2\dot{y}^2\right) + m_1 g x$ where (referring to the diagram with the problem) x
$g(x,y) = 0 = x + 2y + d - l$
increases downward and y increases to the right. The rope has length l and d accounts for all the rope not taken up by x and y. Also from the constraint, we have $\ddot{x} = -2\ddot{y}$. Using the Euler-Lagrange equations with undetermined multiplier gives

$$\begin{matrix} m_1\ddot{x} - m_1 g + \lambda = 0 \\ m_2\ddot{y} + 2\lambda = 0 \end{matrix} \Rightarrow \begin{matrix} -2m_1\ddot{x} + 2m_1 g - 2\lambda = 0 \\ -\frac{m_2\ddot{x}}{2} + 2\lambda = 0 \end{matrix} \Rightarrow \ddot{x} = \frac{4m_1 g}{4m_1 + m_2}$$

Check limits: $m_1 >> m_2$, acceleration is g, : $m_1 << m_2$, acceleration goes to zero.

4. $\frac{GMm}{(a-r)^2}$ ← $\frac{GMm}{(a+r)^2}$ ← FROM M

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→ ← FROM EACHOTHER

$\frac{Gm^2}{(2r)^2}$ $\frac{Gm^2}{(2r)^2}$

FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE LESS THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{G\cancel{Mm}\,m^2}{4r^2}$$

$$\frac{M}{(a^2-r^2)^2}(a+r)^2 - \frac{M}{(a^2-r^2)^2}(a-r)^2 < \frac{m}{2r^2}$$

$$4ar\,\frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\frac{6}{\pi}\frac{M}{a^3} < \rho$$

Príklad 2

3. Current carrying wire (Question and solution from Peter)

a) Take loop of radius r around wire and use Ampere's law:
$\oint \vec{B}\cdot d\vec{l} = {}^{4\pi}\!/_{c}\, I \Rightarrow \vec{B} = B(r)\hat{\varphi} = \frac{2I}{rc}\hat{\varphi} \Rightarrow \vec{F} = q\frac{\vec{v}}{c}\times\vec{B} = \frac{2Ive}{rc^2}\hat{\rho}$ where $\hat{\rho}$ is radial direction in cylindrical coordinates.

b) If F' moves along wire with v, then particle is at rest in F' and the Lorentz force is zero. Transform force from F:
$F_\perp = \frac{dp_\perp}{dt} = \frac{dp'_\perp}{\gamma dt' - \beta\gamma dx'} = \frac{{dp'_\perp}/{dt'}}{\gamma + \beta\gamma\, {dx'}/{dt'}} = \frac{F'_\perp}{\gamma} \Rightarrow F'_{\perp=} = \gamma\frac{2Ive}{rc^2}$. Can also do problem by transforming currents and charge densities.

Príklad 3

a. In order for the masses to collide, the total angular momentum of the system must be zero, which only occurs if $v_0 = 0$.

b. In this case, the masses undergo uniform circular motion with radius $\frac{l}{2}$ and speed v_0, so that

$$\frac{Gm^2}{l^2} = \frac{m{v_0}^2}{\frac{l}{2}}$$

$$\frac{Gm}{{v_0}^2 l} = 2$$

c. The masses follow closed orbits if they do not have enough energy to escape, *i.e.* if the total energy of the system is negative. The total energy of the system is

$$2\cdot\frac{1}{2}m{v_0}^2 - \frac{Gm^2}{l}$$

so that the condition required is

$$m{v_0}^2 - \frac{Gm^2}{l} < 0$$

$$\frac{Gm}{{v_0}^2 l} > 1$$

d. Note that the masses will always move symmetrically about the center of mass. Thus, in order to be at minimum separation, their velocities must be perpendicular to the line joining them (and will be oppositely directed). Let the minimum separation be d, and let the speed of each mass at minimum separation be v.

$$L = 2mv\frac{d}{2} = mvd$$

The initial angular momentum is likewise $mv_0 l$, and so by conservation of angular momentum

$$mvd = mv_0 l$$

$$v = v_0 \frac{l}{d}$$

By conservation of energy

$$2 \cdot \frac{1}{2} m {v_0}^2 - \frac{Gm^2}{l} = 2 \cdot \frac{1}{2} m v^2 - \frac{Gm^2}{d}$$

$${v_0}^2 - \frac{Gm}{l} = v^2 - \frac{Gm}{d}$$

Combining these,

$${v_0}^2 - \frac{Gm}{l} = {v_0}^2 \frac{l^2}{d^2} - \frac{Gm}{d}$$

$$\left(1 - \frac{Gm}{{v_0}^2 l}\right)\left(\frac{d}{l}\right)^2 + \frac{Gm}{{v_0}^2 l}\left(\frac{d}{l}\right) - 1 = 0$$

$$\left(\frac{d}{l} - 1\right)\left(\left(1 - \frac{Gm}{{v_0}^2 l}\right)\frac{d}{l} + 1\right) = 0$$

so that

$$d = l \quad \text{or} \quad d = \frac{l}{\frac{Gm}{{v_0}^2 l} - 1}$$

The second root is only sensible if $\frac{Gm}{{v_0}^2 l} > 1$, and is only smaller than the first if $\frac{Gm}{{v_0}^2 l} > 2$. (Note that both of these results make sense in light of the previous ones.) Thus the minimum separation is l if $\frac{Gm}{{v_0}^2 l} \leq 2$ and $\frac{l}{\frac{Gm}{{v_0}^2 l} - 1}$ otherwise.

Príklad 4

Answer:

$$C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT}$$

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don't have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by x balances pressure

$$pS = kx \qquad pV = pSx = kx^2 = \nu RT$$

and using this connection between x and T we get

$$C = \frac{3}{2}\nu R + kx\frac{dx}{dT} = \frac{3}{2}\nu R + kx\frac{1}{2kx}\nu R = 2\nu R \qquad \boxed{C = 2\frac{p_0 V_0}{T_0}}$$