METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto24 – Príklady 1

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Cvičenie 29.2.2024

Príklad 1

A… The average speed is distance divided by time. The truck moves at constant rate from the start to end position while the car initially moves "backward" and then forward to the same ending location of the truck. Consequently, the car travels a greater distance than the truck and has a higher average speed.
C… The slope of the position vs. time graph. The car moves faster than the truck initially and eventually come to rest, which means that at some point, the speed of the car and truck are the same. From rest, the car accelerates and catches up to the truck by eventually moving faster. This means that the car and truck have the same speed again before time T. Finally, for times $T < t < 2T$, the car slows down close to rest and the truck catches up. Hence, the car's speed is again equal to that of the truck at some time. This makes C the correct answer.

Príklad 2

Príklad 3

Príklad 4

Assume that q is negative and Q is positive.

(a) Apply Gauss's Law to a spherical shell of radius r where $r < R$. Then,

$$E 4\pi r^2 = \frac{4\pi\rho r^3}{\epsilon_0}$$

where the charge density $\rho = \frac{3Q}{4\pi R^3}$

Solving for the Electric Field at a distance r from the center, we find

$$E = \frac{\rho r}{3\epsilon_0}$$

We now find the potential difference between the center of the sphere and the outer boundary of the charged cloud.

$$\Delta V = -\int_0^R E dr$$

Substituting in our expression for E, we now find

$$\Delta V = -\frac{\rho R^2}{6\epsilon_0}$$

Using conservation of energy,

$$K_{lost} = U_{gained}$$

$$K_0 = -q\Delta V = \frac{-qQ}{8\pi R\epsilon_0}$$

(b) Apply Newton's Second Law to the object of charge $-q$ when it is located a distance r away from the center of the sphere.

$$F_{net} = ma$$

$$-(-q)E = m\frac{d^2r}{dt^2}$$

$$\frac{d^2r}{dt^2} = -\frac{-q\rho r}{3\epsilon_0 m}$$

We recognize that this is the differential equation for simple harmonic motion

$$\frac{d^2r}{dt^2} = -\omega^2 r$$

where $\omega = \sqrt{\frac{-q\rho}{3\epsilon_0 m}}$.

Since the charge has the minimum kinetic energy needed to reach the surface, the trip from the center of the sphere to the outer boundary is one-fourth of a cycle of SHM.

Therefore,

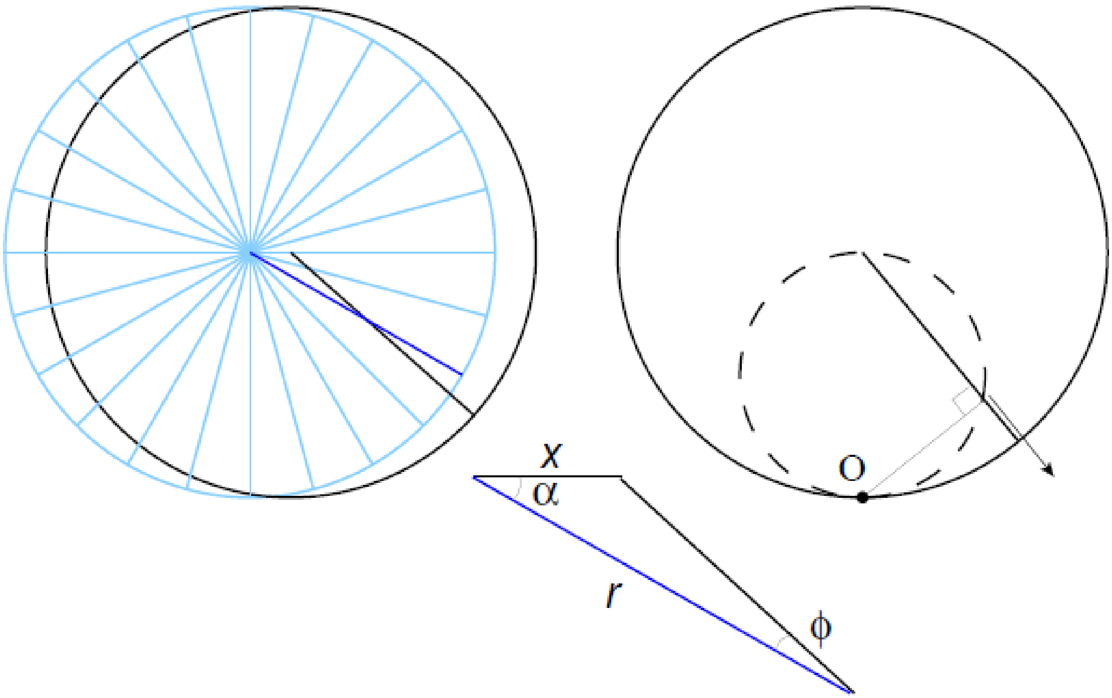
$$t = \frac{T}{4} = \frac{\pi}{2\omega} = \frac{\pi}{2}\sqrt{\frac{3\epsilon_0 m}{-q\rho}}$$

.

Substituting, in for rho, we find

$$t = \frac{\pi}{2}\sqrt{\frac{4\pi\epsilon_0 m R^3}{-qQ}}$$

Príklad 5



As the wheel rolls to the right, it rotates clockwise. A spoke shifts to the right by distance x and rotates by angle ϕ that are related

$$\phi = \frac{x}{R}$$

where R is the radius of the wheel. We want to find the points of the wheel that does not seem to move. This is the intersection points of the spoke with itself at a slightly later time ($\phi \to 0$), from the Law of Sines:

$$\frac{x}{\sin\phi} = \frac{r}{\sin(\pi - \alpha - \phi)} \qquad \Rightarrow \qquad r(\alpha) = \frac{R\phi}{\sin\phi}\sin(\alpha + \phi) = \boxed{R\sin\alpha}$$

that represents a circle of radius $R/2$ centered between the center of the wheel and the instanteneous rotation point O.

Solution 2: if you think about point of instanteneous rotation O, all the elements of the wheel are circling around it. The spoke elements that seem stationary are those that are under this rotation shift along the spoke itself! This means (figure on the right) that the angle between the spoke and the line that connects O and the element of the spoke must be 90 degrees. The geometrical figure that correspond to such a configuration is a circle!

Príklad 6

We assume there are no external forces acting on the rocket, and we choose a closed system so Newton's 2nd Law applies. Then, part 1 is simply a conservation of linear momentum problem.

First, we must find the velocity of the rocket. At a time t, let the instantaneous mass of the rocket be m, and its velocity be v relative to some inertial reference frame. Then, during a time dt, a mass dm' is ejected with speed $-u$ with respect to the rocket. So, conservation of linear momentum gives:

$$\begin{aligned} p(t) &= p(t+dt) \\ mv &= (m-dm')(v+dv)+dm'(v-u) \\ &\Rightarrow \; mdv = udm' \\ &\Rightarrow \; dv = -u\frac{dm}{m} \end{aligned}$$

using $dm = -dm'$ and dropping the product of differentials.

Let the initial mass be m_0 and the initial speed be 0. Then, integrating the above equation gives:

$$\begin{aligned} \int_0^v dv &= -u\int_{m_0}^{m}\frac{dm}{m} \\ v &= u\ln\left(\frac{m_0}{m}\right) \end{aligned}$$

and hence $p = mu\ln\left(\frac{m_0}{m}\right)$.

To find when p is maximized, we take the derivative and set it to 0:

$$\begin{aligned} \dot{p} &= \alpha u\ln\left(\frac{m_0}{m}\right) + mu\left(\frac{m}{m_0}\right)\left(\frac{-m_0\alpha}{m^2}\right) = 0 \\ &\Rightarrow \; \ln\left(\frac{m_0}{m}\right) = 1 \\ &\Rightarrow \; \boxed{m = \frac{m_0}{e}} \end{aligned}$$

Now there is an external force, so

$$F_{ext} = \frac{dp}{dt} \Rightarrow F_{ext}dt = dp = p(t+dt) - p(t)$$

From part 1, we know

$$p(t+dt) - p(t) = mdv + udm$$

We assumed $\dot{m} = \alpha$, and since in a vertical ascent at the surface of the earth, $F_{ext} = -mg$

$$\begin{aligned} F_{ext}dt &= -mgdt = mdv + udm \\ &\Rightarrow \; dv = \left(-g - \frac{\alpha u}{m}\right)dt \end{aligned}$$

Using $\dot{m} = \alpha$ once again to eliminate dt

$$dv = \left(\frac{-g}{\alpha} - \frac{u}{m}\right)dm$$

To ensure $dv > 0$ when the engine fires, it is required that $\left(\frac{-g}{\alpha} - \frac{u}{m_0}\right) > 0$, and hence

$$\boxed{u > -\frac{gm_0}{\alpha}}$$

To find v at any time shortly after liftoff (so we can assume the acceleration due to gravity is well approximated to be g), we must integrate the above equation for dv, as in part 1

$$\begin{aligned} \int_0^v dv &= \int_{m_0}^{m}\left(\frac{-g}{\alpha} - \frac{u}{m}\right)dm \\ v &= -\frac{g}{\alpha}(m - m_0) + u\ln\left(\frac{m_0}{m}\right) \end{aligned}$$

Integrating $dm = \alpha dt$ gives $m - m_0 = \alpha t$, and hence we obtain

$$\boxed{v = -gt + u\ln\left(\frac{m_0}{m_0 + \alpha t}\right)}$$