

METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto24 – Príklady 2

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Príklad 1

[28.] Kľúčovým je uvedenie si, že nás nezaujíma priebeh pohybu telieska, ale iba počiatočný a koncový stav a tiež fakt, že v gravitačnom poli sa celková energia telesa zachováva. Preto je zákon zachovania energie presne to, čo potrebujeme. Tak si ho pre našu situáciu zapíšeme:

$$-\kappa \frac{M_1 m}{R_1} - \kappa \frac{M_2 m}{2R_1 + R_2} = \frac{1}{2} m v^2 - \kappa \frac{M_2 m}{R_2} - \kappa \frac{M_1 m}{R_1 + 2R_2}$$

Odtiaľto už ľahko dostaneme hľadanú rýchlosť telieska:

$$v = 2 \sqrt{\kappa \left(M_2 \frac{R_1}{R_2(2R_1 + R_2)} - M_1 \frac{R_2}{R_1(R_1 + 2R_2)} \right)}$$

Príklad 2

Príklad 3

$$E = 0 = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + mgy$$

$$y = -ax^n \Rightarrow \dot{y} = -nax^{n-1} \dot{x}$$

$$\Rightarrow \dot{x}^2 = \frac{2gax^n}{1+n^2a^2x^{2(n-1)}}$$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$$2\cancel{x} \ddot{x} = \frac{\partial}{\partial t} \left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}} \right) = \cancel{x} \frac{\partial}{\partial x} \left(\frac{2gax^n}{1+n^2a^2x^{2(n-2)}} \right) = 0$$

$$\Rightarrow \frac{nx^{n-1}}{1+n^2a^2x^{2(n-1)}} - \frac{2(n-1)n^2a^2x^{3n-3}}{(1+n^2a^2x^{2(n-2)})^2} = 0$$

$$nx^{n-1} + n^3a^2x^{3n-3} - 2(n-1)n^2a^2x^{3n-3} = 0$$

$$\Rightarrow a^2x^{2n-2} = \frac{1}{n(n-2)}. \text{ Real finite solution only for } n > 2.$$

Príklad 4

a) The effective potential per unit mass is

$$V_{\text{eff}}(r) = \epsilon - \frac{\dot{r}^2}{2} = \frac{\ell^2}{2r^2} - \frac{K^2}{r^4},$$

where ϵ is the energy per unit mass and ℓ is the angular momentum per unit mass. The circular orbit has $dV_{\text{eff}}/dr = 0$ or $r_{\text{circ}} = 2K/\ell$. Hence the period is $P = 2\pi r^2/\ell = \pi r^3/K$. V_{eff} has a maximum at r_{circ} and so the equilibrium is unstable.

b) A spaceship with $\epsilon < V_{\text{circ}} = \ell^4/16K^2$ will be repelled by the effective potential and so capture will be avoided if

$$\frac{1}{2}v^2 = \epsilon < V_{\text{circ}} = \frac{\ell^4}{16K^2} = \frac{v^4b^4}{16K^2}$$

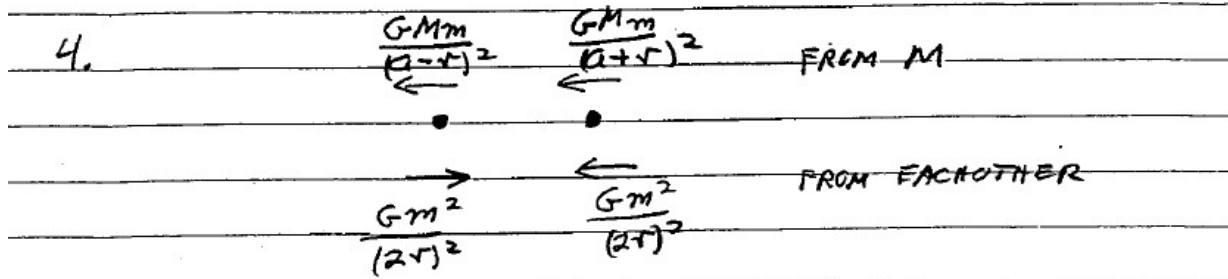
where b is the impact parameter. Hence

$$b > \left(\frac{8K^2}{v^2} \right)^{1/4}$$

to avoid capture.

- c) Just treat the object as having a scattering cross-section of b . Then the mass capture rate is $\pi b^2 \times \rho v = 2\sqrt{2}\pi K\rho$.

Príklad 5



FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE LESS THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{GMm}{4r^2}$$

$$\frac{M}{(a^2-r^2)^2} (a+r)^2 - \frac{m}{(a^2-r^2)^2} (a-r)^2 < \frac{m}{2r^2}$$

$$4ar \frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\frac{6}{\pi} \frac{M}{a^3} < \rho$$

Příklad 6



magnetic fields using Ampère's Law

$$\begin{aligned}
 r > b & \quad \vec{B} = 0 \\
 a < r < b & \quad \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I \Rightarrow 2\pi s B = \mu_0 I \\
 & \quad \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi} \\
 r < a & \quad 2\pi s B = \mu_0 I \frac{s^2}{a^2} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi} \frac{s^2}{a^2} \hat{\phi}
 \end{aligned}$$

$$W = \frac{1}{2\mu_0} \iiint B^2 dV, \text{ so per unit length}$$

$$\begin{aligned}
 \frac{W}{\ell} &= \frac{\mu_0}{2} \frac{I^2}{(2\pi)^2} 2\pi \left\{ \int_0^a \frac{s^2}{a^4} s ds + \int_a^b \frac{1}{s^2} s ds \right\} \\
 &= \frac{\mu_0}{4\pi} I^2 \left\{ \left[\frac{1}{4} \frac{s^4}{a^4} \right]_0^a + \left[\ln s \right]_a^b \right\} \\
 &= \frac{\mu_0}{16\pi} I^2 + \frac{\mu_0}{4\pi} \ln \frac{b}{a} I^2
 \end{aligned}$$

Now use $W = \frac{1}{2} \mathcal{L} I^2$, so

$$\mathcal{L} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

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 vanishes for
 cylindrical shell