METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto24 – Príklady 3

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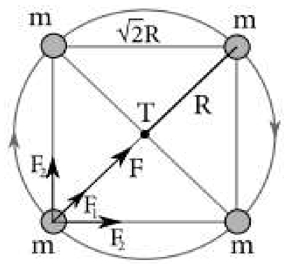
Príklad 1

Planéty na seba navzájom pôsobia gravitačnými silami. Situácia je zjavne úplne stredovo súmerná, takže môžeme spočítať, aká sila pôsobí na ľubovoľnú z nich.



Protiľahlá planéta sa nachádza vo vzdialenosti $2R$, preto pôsobí silou veľkosti $F_1 = \frac{Gm^2}{4R^2}$. Priľahlé planéty sa nachádzajú vo vzdialenosti $\sqrt{2}R$, preto budú pôsobiť silami veľkosti $F_2 = \frac{Gm^2}{2R^2}$. Zo symetrie úlohy je zrejmé, že výslednica síl od dvoch priľahlých planét bude smerovať do stredu, preto si ich rozložme do dostredného smeru a smeru naň kolmého. Kolmé zložky sa vybijú a prežijú iba dostredné zložky $\frac{1}{\sqrt{2}}F_2$.

Výsledná sila pôsobiaca na planétu teda bude

$$F = F_1 + 2 \cdot \frac{1}{\sqrt{2}} F_2 = \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) \frac{Gm^2}{R^2}.$$

Táto sila spôsobuje pohyb po kružnici, čiže je dostredivou silou $F = m\omega^2 R$. Z rovnosti dostredivej a výslednej gravitačnej sily dostávame $\omega = \sqrt{\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) \frac{Gm}{R^3}}$. Z definície uhlovej rýchlosti $\omega = \frac{2\pi}{T}$ dopočítame periódu obehu planét

$$T = \frac{4\pi}{\sqrt{2\sqrt{2}+1}} \sqrt{\frac{R^3}{Gm}}.$$

Príklad 2

Príklad 3

One can express the net radiative transfer, I_{net}, in terms of the left and right portions of the system.

$$I_R = \epsilon_L \sigma T_L^4 + (1 - \epsilon_L) I_L$$

$$I_L = \epsilon_R \sigma T_R^4 + (1 - \epsilon_R) I_R$$

$$I_{\text{net}} = I_R - I_L$$

By looking at the quantity $\epsilon_R I_R - \epsilon_L I_L$ and isolating $I_R - I_L$, one finds...

$$\boxed{I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{\epsilon_R + \epsilon_L - \epsilon_L \epsilon_R}}$$

For the infinite series approach, consider the contribution from each sequence. That is.

$$I'_R = \epsilon_R \epsilon_L \sigma T_L^4 (1 + (1-\epsilon_R)(1-\epsilon_L) + ((1-\epsilon_R)(1-\epsilon_L))^2 + ...)$$

and the same for I'_L. The infinite series term is just the geometric series with a value $(1-(1-\epsilon_R)(1-\epsilon_L))^{-1}$. Therefore, the net effect is

$$I_{\text{net}} = \frac{\epsilon_R \epsilon_L \sigma (T_L^4 - T_R^4)}{1-(1-\epsilon_R)(1-\epsilon_L)}$$

Which is the same as above.

Príklad 4

ANGULAR MOMENTUM CONSERVATION:

INITIALLY $L = |\vec{r} \times \vec{p}| = mvr|\sin\theta| = mvb$

AT PERIGEE $L = m v_p r_p$

$\therefore v_p = vb/r_p$

ENERGY CONSERVATION: $KE + \frac{GMm}{r_p} \simeq 0$

$\therefore \frac{1}{2} m v_p^2 = \frac{GMm}{r_p}$

SUBSTITUTE FOR v_p: $\frac{v^2 b^2}{2 r_p^2} = \frac{GM}{r_p}$

HENCE: $r_p = \frac{v^2 b^2}{2GM}$

Príklad 5

The linearity of Maxwell's equations allows us to find the magnetic field as a sum of two magnetic fields produced by two currents: A current with density

$$j_b = \frac{I}{\pi(b^2-a^2)} \tag{1}$$

carried by the cylinder of radius b and a current with density

$$j_a = -j_b \tag{2}$$

carried by a cylinder of radius a. The sum of these two currents gives the current distribution in the considered structure. From Ampere's circuital law $\oint \mathbf{H} \cdot d\mathbf{l} = \frac{4\pi}{c} \int \mathbf{j} \cdot d\mathbf{S}$ one finds that the current carried by the cylinder of radius b produces a magnetic field at the center of the hole

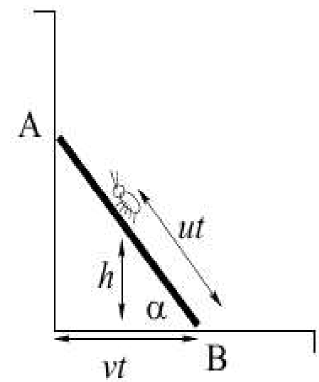
$$H_b = \frac{2Id}{c(b^2 - a^2)}, \tag{3}$$

while the current carried by the cylinder of radius a produces no magnetic field at the center of the hole, $H_a = 0$. Therefore, the magnetic field at the center of the hole is

$$H = \frac{2Id}{c(b^2 - a^2)}. \tag{4}$$

Príklad 6

At time t, lower end of the stick will be distance vt from the wall, making angle $\cos\alpha = vt/L$ with the floor, and the bug progressed to a point ut from this end (along the hypotenuse). The height above the floor will be



$$h(t) = ut \sin\alpha = ut\sqrt{1 - \frac{v^2t^2}{L^2}}$$

Maximal height will be reached at time $t = \frac{L}{v\sqrt{2}}$, when the stick makes angle 45 deg with the floor, and it will be

$$\boxed{h_{max} = \frac{Lu}{2v}}$$

If the bug is fast, and it reaches the end of the stick (in time $t = L/u$) before the stick makes 45-angle, then the maximal height is

$$\boxed{h_{max} = L\sqrt{1 - \frac{v^2}{u^2}}}$$