

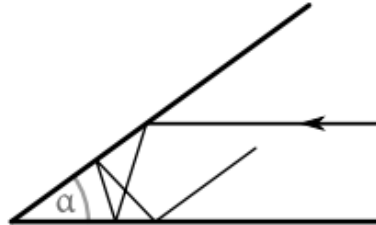
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto24 – Príklady 6

Cvičenie 25. 4. 2024

Príklad 1

17 Enka vzala dve zrkadielka a postavila ich tak, aby zvierali uhol α . Potom medzi ne zasvietila laserom tak, že lúč bol rovnobežný s jedným zo zrkadiel. Všimla si, že naspäť sa lúč spomedzi zrkadiel vracia presne pozdĺž druhého zrkadla.

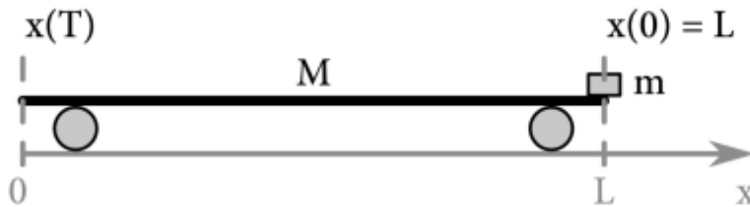
Aké všetky uhly mohli zrkadlá zvierat?



Príklad 2

Ak viete, že v nasledujúcich úlohách je práve jeden z ponúkaných výsledkov správny, nájdite ho bez toho, aby ste úlohu počítali.

Na skateboarde hmotnosti M a dĺžky L sa na prednom konci nachádza kameň hmotnosti m . Medzi kameňom a skateboardom však pôsobí trenie s koeficientom μ . Keď postrčíme skateboard nohou, udelíme mu rýchlosť v v pozdĺžnom smere. Aká je najväčšia možná rýchlosť postrčenia, aby sa kameň nezošmykol zo zadnej strany skateboardu?



Two columns:

$$1. v = \sqrt{2\mu g L \left(\frac{m+M}{M}\right)}$$

$$4. v = \sqrt{\frac{2g L}{\mu} \left(\frac{m+M}{M}\right)}$$

$$2. v = \sqrt{\mu g L \left(\frac{M}{M+m}\right)}$$

$$5. v = \left(\frac{m+M}{M}\right) \sqrt{g L \left(1 + \frac{m\mu}{M}\right)}$$

$$3. v = \sqrt{g L \left(\frac{2m+M}{M\mu}\right)}$$

$$6. v = \sqrt{\frac{\mu g}{L} \left(\frac{m+M}{3M}\right)}$$

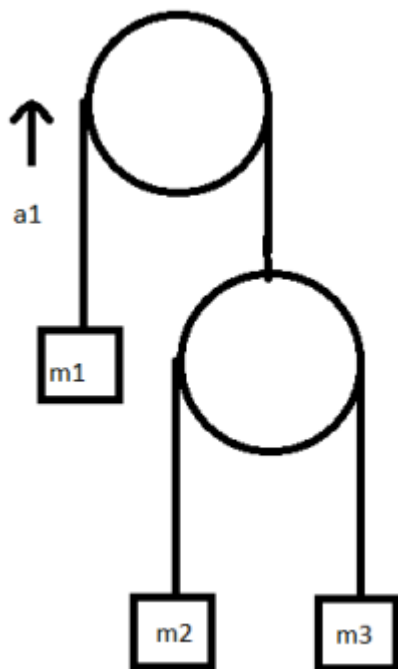
Dve gule s hmotnosťami m a M sa zrazia celne rýchlosťami veľkosti v a V . Aké budú rýchlosti guľ po zrážke?

$$a) V' = \frac{2m}{m+M} v - \frac{m+M}{m+M} V, v' = \frac{2M}{M+m} V - \frac{M+m}{M+m} v$$

$$b) V' = \frac{2m}{m+M} v + \frac{m-M}{m+M} V, v' = \frac{2M}{M+m} V + \frac{M+m}{M+m} v$$

$$c) V' = \frac{2m}{m+M} v - \frac{m-M}{m+M} V, v' = \frac{2M}{M+m} V - \frac{M+m}{M+m} v$$

$$d) V' = \frac{m-M}{m+M} v - \frac{2m}{m+M} V, v' = \frac{M-m}{M+m} V - \frac{2M}{M+m} v$$



$$\text{a) } a_1 = \frac{4m_2m_3 - m_1(m_2 + 1)}{4m_2m_3 + m_1(m_2 + 1)} g$$

$$\text{b) } a_1 = \frac{m_1(m_2 + m_3)}{4m_2m_3 + m_1(m_2 + m_3)} g$$

$$\text{c) } a_1 = \frac{4m_2m_3 - m_1(m_2 + m_3)}{4m_2m_3 + m_1(m_2 + m_3)} g$$

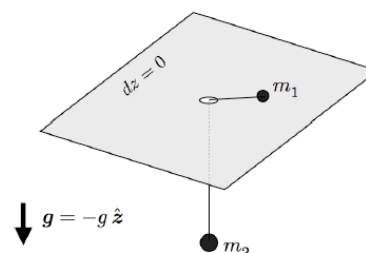
$$\text{d) } a_1 = \frac{4m_2m_3 + m_1(m_2 + m_3)}{4m_2m_3 - m_1(m_2 + m_3)} g$$

$$\text{e) } a_1 = \frac{4m_2m_3}{4m_2m_3 + m_1(m_2 + m_3)} g$$

Príklad 3

PROBLEM: An inextensible massless string of length ℓ passes through a hole in a horizontal table. A point mass m_1 on one end of the string moves frictionlessly along the table (*i.e.* with two degrees of freedom), and another point mass m_2 dangles vertically from the other end. (See the sketch below.)

- Write the Lagrangian for this system.
- Under what conditions will the hanging mass remain stationary?
- Starting from the situation in part (b), the hanging mass is pulled down slightly and then released. State clearly what is conserved during this process.
- Compute the subsequent motion of the hanging mass.



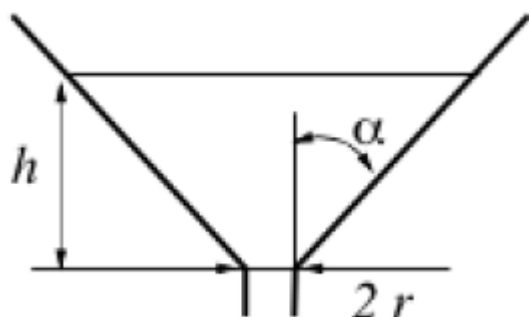
Príklad 4

You are driving at a constant speed of $v = 30 \text{ m/s}$, always in the NW direction. You are driving on a horizontal sheet of ice on the Arctic Ocean, with coefficient of friction $\mu = 0.1$. At what distance R from the N pole do you start to skid? Take $g = 9.8 \text{ m/s}^2$.

Príklad 5

The stem of the conical funnel, see figure, is initially closed. The funnel is filled with an incompressible liquid (mass density ρ) up to the level h . At $t = 0$, the funnel is opened and the liquid begins to flow out.

- Find the flux, J , through the stem as a function of h .
- Find the velocity of the upper surface of the liquid dh/dt as the function of the flux.
- Write down a differential equation whose solution would yield $h(t)$.
- Solve the differential equation to obtain an explicit form of $h(t)$.



Příklad 6

Consider a cylindrical capacitor of length L . The capacitor consists of an inner conducting wire of radius a and an outer conducting shell of radius b . The space between ($a < r < b$) is filled with a non-conducting material which has a dielectric constant ϵ . In all cases, neglect any end effects or fringing fields.

- What is the value of the electric field as a function of the radial position, r , when the capacitor has charge Q on it?
- What is the capacitance?
- Now suppose the dielectric is pulled out partially while the capacitor is connected to a battery of constant potential V . What is the force required to hold the dielectric in this position?