METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 2 zima24 – Príklady 3

VZOROVÉ RIEŠENIA

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Príklad 1

$$L = \frac{m}{2}\left[\dot{x}^2 + \left(\frac{l}{2}\right)^2 \cos^2\theta\,\dot{\theta}^2\right] + \frac{ml^2}{24}\dot{\theta}^2 - \frac{mgl}{2}\sin\theta$$

$$\dot{x} = \text{const.} = 0 \quad , \quad x = 0$$

$$H = \left(\frac{m}{2}\frac{l^2}{4}\cos^2\theta + \frac{ml^2}{24}\right)\dot{\theta}^2 + \frac{mgl}{2}\sin\theta = \frac{mgl}{2}\sin\theta_0$$

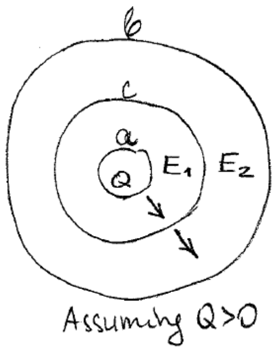
$$\left(\frac{d\theta}{dt}\right)^2 = \frac{(g/l)\,\frac{1}{2}\,(\sin\theta_0 - \sin\theta)}{\frac{1}{8}\cos^2\theta + \frac{1}{24}}$$

$$\sqrt{g/l}\; t = \int_0^{\theta_0} \sqrt{\frac{\frac{1}{4}\cos^2\theta + \frac{1}{12}}{\sin\theta_0 - \sin\theta}}\, d\theta$$

$$\Delta X_{end} = \frac{l}{2} - \frac{l}{2}\cos\theta_0$$

Príklad 2

3.



(a) Gauss law:

$$D = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_1 = \frac{D}{\varepsilon_1} = \frac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2}$$

$$E_2 = \frac{D}{\varepsilon_2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2}$$

(b)

$$\phi_a - \phi_c = \int_a^c dr\, E_1 = \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right)$$

$$\phi_b - \phi_c = \int_c^b dr\, E_2 = \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)$$

$$\phi_a - \phi_b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]$$

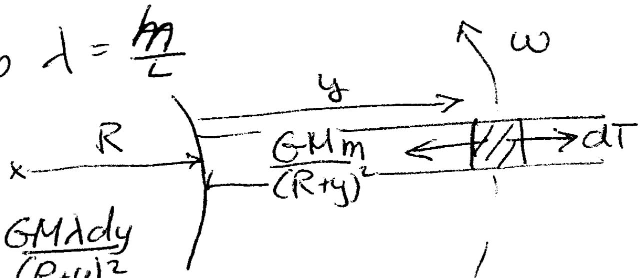
$$C = \frac{Q}{\phi_a - \phi_b} = 4\pi\varepsilon_0\left[\frac{1}{\varepsilon_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]^{-1}$$

(c) Gauss law:

$$\sigma = \varepsilon_0 (E_2 - E_1)\Big|_{z=0} = \frac{Q}{4\pi\varepsilon_0 S^2}\left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1}\right) \qquad V$$

Príklad 3

A. The density is $\lambda = \frac{m}{L}$



$$\frac{GMm}{(R+y)^2} + dT = m\omega^2 (R+y)$$

$$\therefore dT = \lambda dy \omega^2 (R+y) - \frac{GM\lambda dy}{(R+y)^2}$$

$$T(y) = \frac{GM\lambda}{(R+y)} + \frac{\lambda\omega^2}{2}(R+y)^2 \Big|_y^L$$

$$\therefore T(y) = \frac{GM\lambda}{R+L} - \frac{GM\lambda}{R+y} + \frac{\lambda\omega^2 (R+L)^2}{2} - \frac{\lambda\omega^2 (R+y)^2}{2}$$

$$= GM\lambda\left(\frac{1}{R+L} - \frac{1}{R+y}\right) + \frac{\lambda\omega^2}{2}\left[R^2 + 2LR + L^2 - R^2 - 2Ry - y^2\right]$$

$$= GM\lambda\left(\frac{y-L}{(R+L)(R+y)}\right) + \frac{\lambda\omega^2}{2}\left(2R(L-y) + L^2 - y^2\right)$$

$$\boxed{T(y) = (L-y)\lambda\left[\frac{-GM}{(R+L)(R+y)} + \frac{\omega^2}{2}(2R+L+y)\right]}$$

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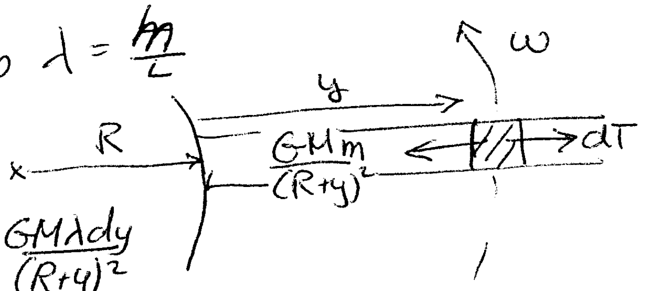

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