METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH zima24 – Príklady 4

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Príklad 1

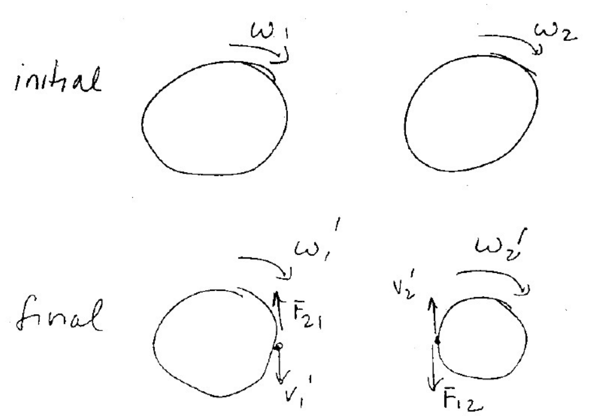
- Introduce the tension T in the rope and the force F which the mass M_h exerts to the right on the sphere. Use X, Y and x for the laboratory coordinates of M_h, M_v and m respectively.
- Write down equations for the acceleration of the cm of each mass:

$$\begin{aligned} M_h\ddot{X} &= T - F \\ m\ddot{x} &= F \\ M_v\ddot{Y} &= T - M_v g \end{aligned}$$

- Let θ represent the angular orientation of the sphere (increasing with clockwise motion) and write the equation for the angular acceleration of the sphere and the relation between $\ddot{\theta}$, $\ddot{a}$ and $\ddot{X}$:

$$\begin{aligned} \frac{2}{5}mR^2\ddot{\theta} &= -FR \\ \ddot{a} &= \ddot{X} + R\ddot{\theta} \end{aligned}$$

- These five equations can then be solved for T, F, $\ddot{\theta}$, $\ddot{X}$, $\ddot{Y}$,



Since the tangential final speeds must be the same (but opposite directions)

$$\omega_1' r_1 = -\omega_2' r_2 \quad (5)$$

$$\Rightarrow \quad \omega_2' = -\frac{r_1}{r_2}\omega_1'$$

Since the integral of the torque equals the change in angular momentum

$$I_1(\omega_1' - \omega_1) = -\int |r_1||F_{21}|dt \quad (5)$$

and $I_2(\omega_2' - \omega_2) = -\int |r_2||F_{12}|dt$ but $|F_{21}| = |F_{12}|$ from Newtons 3rd law

$$\therefore \quad \frac{I_1}{r_1}(\omega_1' - \omega_1) = \frac{I_2}{r_2}(\omega_2' - \omega_2)$$

Subst in for $\omega_2' = -\frac{r_1}{r_2}\omega_1'$

$$\therefore \frac{I_1}{r_1}\omega_1' + \frac{I_2}{r_2}\cdot\frac{r_1}{r_2}\omega_1' = \frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2$$

$$\omega_1' = \frac{\frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2}{\frac{I_1}{r_1} + I_2\frac{r_1}{r_2}} = \frac{\frac{L_1}{r_1} - \frac{L_2}{r_2}}{\frac{I_1}{r_1} + \frac{I_2 r_1}{r_2}}$$

with $I = \frac{1}{2}mr^2$ and some algebra (2)

$$\boxed{\omega_1' = \frac{m_1 r_1 \omega_1 - m_2 r_2 \omega_2}{(m_1+m_2)r_1}} \quad (3)$$

Príklad 2

Answer:

$$C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT}$$

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don't have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by x balances pressure

$$pS = kx \qquad pV = pSx = kx^2 = \nu RT$$

and using this connection between x and T we get

$$C = \frac{3}{2}\nu R + kx\frac{dx}{dT} = \frac{3}{2}\nu R + kx\frac{1}{2kx}\nu R = 2\nu R \qquad \boxed{C = 2\frac{p_0 V_0}{T_0}}$$

Príklad 3

a. In order for the masses to collide, the total angular momentum of the system must be zero, which only occurs if $v_0 = 0$.

b. In this case, the masses undergo uniform circular motion with radius $\frac{l}{2}$ and speed v_0, so that

$$\frac{Gm^2}{l^2} = \frac{m{v_0}^2}{\frac{l}{2}}$$

$$\frac{Gm}{{v_0}^2 l} = 2$$

c. The masses follow closed orbits if they do not have enough energy to escape, *i.e.* if the total energy of the system is negative. The total energy of the system is

$$2\cdot\frac{1}{2}m{v_0}^2 - \frac{Gm^2}{l}$$

so that the condition required is

$$m{v_0}^2 - \frac{Gm^2}{l} < 0$$

$$\frac{Gm}{{v_0}^2 l} > 1$$

d. Note that the masses will always move symmetrically about the center of mass. Thus, in order to be at minimum separation, their velocities must be perpendicular to the line joining them (and will be oppositely directed). Let the minimum separation be d, and let the speed of each mass at minimum separation be v.

$$L = 2mv\frac{d}{2} = mvd$$

The initial angular momentum is likewise mv_0l, and so by conservation of angular momentum

$$mvd = mv_0l$$

$$v = v_0\frac{l}{d}$$

By conservation of energy

$$2 \cdot \frac{1}{2}m{v_0}^2 - \frac{Gm^2}{l} = 2 \cdot \frac{1}{2}mv^2 - \frac{Gm^2}{d}$$

$${v_0}^2 - \frac{Gm}{l} = v^2 - \frac{Gm}{d}$$

Combining these,

$${v_0}^2 - \frac{Gm}{l} = {v_0}^2\frac{l^2}{d^2} - \frac{Gm}{d}$$

$$\left(1 - \frac{Gm}{{v_0}^2l}\right)\left(\frac{d}{l}\right)^2 + \frac{Gm}{{v_0}^2l}\left(\frac{d}{l}\right) - 1 = 0$$

$$\left(\frac{d}{l} - 1\right)\left(\left(1 - \frac{Gm}{{v_0}^2l}\right)\frac{d}{l} + 1\right) = 0$$

so that

$$d = l \quad \text{or} \quad d = \frac{l}{\frac{Gm}{{v_0}^2l} - 1}$$

The second root is only sensible if $\frac{Gm}{{v_0}^2l} > 1$, and is only smaller than the first if $\frac{Gm}{{v_0}^2l} > 2$. (Note that both of these results make sense in light of the previous ones.) Thus the minimum separation is l if $\frac{Gm}{{v_0}^2l} \leq 2$ and $\frac{l}{\frac{Gm}{{v_0}^2l}-1}$ otherwise.