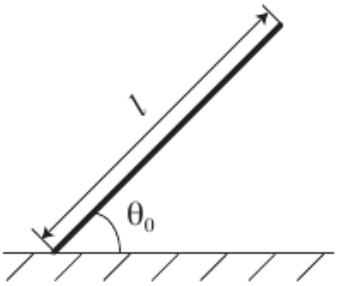
METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 2 zima24 – Príklady 3

Cvičenie 23.10.2024

Príklad 1

PROBLEM: A thin rod of length l is supported at one end by a smooth floor (see figure). The rod is released from a configuration where it makes an angle θ_0 relative to the horizontal. Write down the Lagrangian for this system. Determine how long it takes for the rod to fall to the floor (*the answer in terms of a definite integral will be sufficient.*) Also determine how far the lower end moves during this time.



Príklad 2

A spherical capacitor consists of two concentric conducting spheres of radii a and b. The capacitor is filled with dielectric material whose dielectric constant varies according to:

$$\varepsilon(r) = \begin{cases} \varepsilon_1 = \text{const}, & \text{for } a < r < c \\ \varepsilon_2 = \text{const}, & \text{for } c < r < b \end{cases}$$

The charge on the *inner* conducting sphere is Q.

(a) Find the electric field inside the capacitor as a function of r.

(b) Find the capacitance of this capacitor.

(c) Find the density of the bound charge on the boundary between the dielectric layers at $r = c$.

Príklad 3

A recent Science Times article featured the concept of a "space elevator". This is a free hanging rope in stationary orbit around the earth above the equator. You could send an elevator up this rope to launch objects into space at less cost than required for shuttle flights. Imagine such a rope just reached the earth's surface. Find an expression for the tension in the rope as a function of height, y, off the earth's surface. Assume the rope has length L, and mass m, and that the earth has radius R and mass M and rotates at angular velocity ω. What length, L, allows the rope to hang freely (i.e. without being attached to the earth's surface) ?