METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 1 leto25 – Príklady 2

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Príklad 1

$\boxed{28.}$ Kľúčovým je uvedomenie si, že nás nezaujíma priebeh pohybu telieska, ale iba počiatočný a koncový stav a tiež fakt, že v gravitačnom poli sa celková energia telesa zachováva. Preto je zákon zachovania energie presne to, čo potrebujeme. Tak si ho pre našu sitáciu zapíšeme:

$$-\kappa\frac{M_1 m}{R_1} - \kappa\frac{M_2 m}{2R_1 + R_2} = \frac{1}{2}mv^2 - \kappa\frac{M_2 m}{R_2} - \kappa\frac{M_1 m}{R_1 + 2R2}$$

Odtiaľto už ľahko dostaneme hľadanú rýchlosť telieska:

$$v = 2\sqrt{\kappa\left(M_2\frac{R_1}{R_2(2R_1 + R_2)} - M_1\frac{R_2}{R_1(R_1 + 2R_2)}\right)}$$

Príklad 2

Príklad 3

$E = 0 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy$

$y = -ax^n \;\Rightarrow\; \dot{y} = -nax^{n-1}\dot{x}$

$\Rightarrow\; \dot{x}^2 = \frac{2gax^n}{1+n^2a^2x^{2(n-1)}}$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$2\cancel{\dot{x}}\;\ddot{x} = \frac{\partial}{\partial t}\left(\frac{2gax^n}{1+n^2a^2x^{2(n-1)}}\right) = \cancel{\dot{x}}\;\frac{\partial}{\partial x}\left(\frac{2gax^n}{1+n^2a^2x^{(2n-2)}}\right) = 0$

$\Rightarrow\;\; \frac{nx^{n-1}}{1+n^2a^2x^{2(n-1)}} - \frac{2(n-1)n^2a^2x^{3n-3}}{(1+n^2a^2x^{2n-2})^2} = 0$

$nx^{n-1} + n^3a^2x^{3n-3} - 2(n-1)n^2a^2x^{3n-3} = 0$

$\Rightarrow\;\; a^2x^{2n-2} = \frac{1}{n(n-2)}$. Real finite solution only for $n > 2$.

Príklad 4

a) The effective potential per unit mass is

$$V_{\text{eff}}(r) = \epsilon - \frac{\dot{r}^2}{2} = \frac{\ell^2}{2r^2} - \frac{K^2}{r^4},$$

where ϵ is the energy per unit mass and ℓ is the angular momentum per unit mass. The circular orbit has $dV_{\text{eff}}/dr = 0$ or $r_{\text{circ}} = 2K/\ell$. Hence the period is $P = 2\pi r^2/\ell = \pi r^3/K$. V_{eff} has a maximum at r_{circ} and so the equilibrium is unstable.

b) A spaceship with $\epsilon < V_{\text{circ}} = \ell^4/16K^2$ will be repelled by the effective potential and so capture will be avoided if

$$\frac{1}{2}v^2 = \epsilon < V_{\text{circ}} = \frac{\ell^4}{16K^2} = \frac{v^4b^4}{16K^2}$$

where b is the impact parameter. Hence

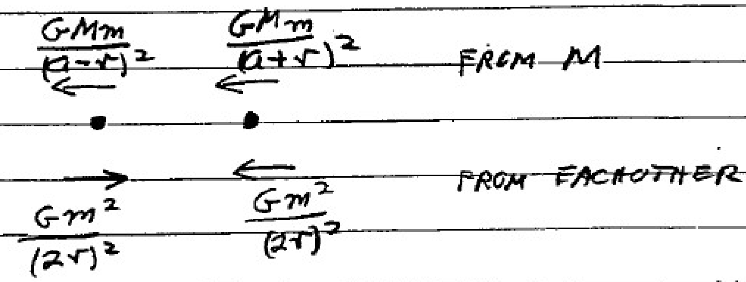
$$b > \left(\frac{8K^2}{v^2}\right)^{1/4}$$

to avoid capture.

c) Just treat the object as having a scattering cross-section of b. Then the mass capture rate is $\pi b^2 \times \rho v = 2\sqrt{2}\pi K \rho$.

Príklad 5

4.



FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE <u>LESS</u> THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{G\cancel{Mm}\,m^2}{4r^2}$$

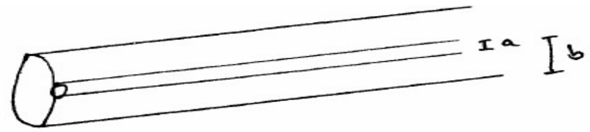
$$\frac{M}{(a^2-r^2)^2}(a+r)^2 - \frac{M}{(a^2-r^2)^2}(a-r)^2 < \frac{m}{2r^2}$$

$$4ar\,\frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\underline{\frac{6}{\pi}\frac{M}{a^3} < \rho}$$

Príklad 6



magnetic fields using Ampère's law

$r > b \qquad \vec{B} = 0$

$a < r < b \qquad \oint \vec{B}\, d\vec{\ell} = \mu_0 I \quad \Rightarrow \quad 2\pi s B = \mu_0 I$

$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi s}\, \hat{\phi}$

$r < a$

$2\pi s B = \mu_0 I \frac{s^2}{a^2} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0}{2\pi} \frac{I s}{a^2}\, \hat{\phi}$

$$W = \frac{1}{2\mu_0} \iiint B^2\, dV$$, so per unit length

$$\frac{W}{\ell} = \frac{\mu_0}{2} \frac{I^2}{(2\pi)^2} 2\pi \left\{ \int_0^a \frac{s^2}{a^4}\, s\, ds + \int_a^b \frac{1}{s^2}\, s\, ds \right\}$$

$$= \frac{\mu_0}{4\pi} I^2 \left\{ \left[\frac{1}{4} \frac{s^4}{a^4} \right]_0^a + \left[\ln s \right]_a^b \right\}$$

$$= \frac{\mu_0}{16\pi} I^2 + \frac{\mu_0}{4\pi} \ln \frac{b}{a}\, I^2$$

Now use $W = \frac{1}{2} L I^2$, so $\quad \frac{L}{\ell} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln b/a$

vanishes for cylindrical shell