METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 2 zima23 – Príklady 2

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Príklad 1

(a) Let v_0 denote the asymptotic common speed.

In a given time interval Δt, the cluster collides with $v_0\Delta t/d$ further beads, which increases its mass by $\Delta m = mv_0\Delta t/d$ and its momentum by $\Delta p = v_0\Delta m = mv_0^2\Delta t/d$. According to Newton's law of motion,

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_0^2}{d},$$

which yields $v_0 = \sqrt{Fd/m}$ for the ultimate speed in the case of inelastic collisions.

(b) In an elastic collision between two equal mass bodies with one of them initially at rest, their velocities are exchanged. The body initially moving with velocity v stops, while the second one moves away with velocity v.

The leftmost bead accelerates uniformly and reaches a speed of

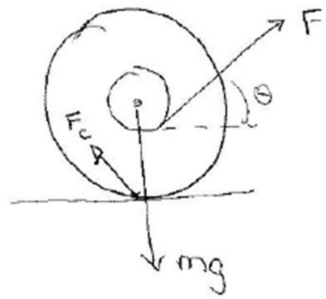
$$v_1 = \sqrt{\frac{2Fd}{m}} = \sqrt{2}v_0$$

before the first (elastic) collision takes place. It then transfers its speed to the second bead and stops, after which it starts accelerating again as a result of the external force. The second bead moves at a constant speed v_1, collides with the third bead and stops. The third and subsequent beads behave similarly, and a 'shock wave' propagates forward at speed v_1.

Meanwhile, the leftmost bead is again accelerated to speed v_1, collides with the second bead, which is now at rest, and the process is repeated, thus starting a new 'shock wave'. The speed of the leftmost bead varies uniformly from zero to v_1, its average value is $v_1/2 = v_0/\sqrt{2} = \sqrt{Fd/(2m)}$.

Príklad 2

2.



y: $\Sigma F_y = F_{cy} + F\sin\theta - mg = 0$

x: $\Sigma F_y = -F_{cx} + F\cos\theta = MR\frac{d\omega}{dt}$

τ about CM $\Sigma \vec{\tau} = +F_{cx}R - Fr = I_0 \frac{d\omega}{dt}$

$$F_{cx} = F\cos\theta - MR\frac{d\omega}{dt}$$

$$\left(F\cos\theta - MR\frac{d\omega}{dt}\right)R - Fr = I_0\frac{d\omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{FR\cos\theta - Fr}{I_0 + MR^2}$$

Forward motion occurs when $\frac{d\omega}{dt} > 0$ if

$FR\cos\theta > Fr$ $\quad \cos\theta > \frac{r}{R}$

Backward motion occurs when $\cos\theta < \frac{r}{R}$

So. $\cos\theta_c = \frac{r}{R}$

Príklad 3

Solution. The boat ends up where it started! The mass of the man does not matter, nor do the length and the mass of the boat, nor does the magnitude of the drag coefficient k. The problem requires *no* data!

A description of the motion. (A precise solution is given in the next paragraph.) When the person starts walking right, the boat starts moving left (Figure 14.5). Hence drag force points right, and by Newton's second law the center of mass

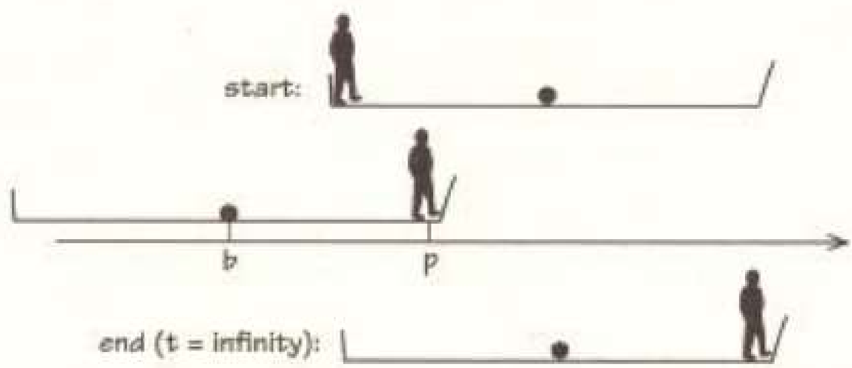


Figure 14.5. The boat eventually approaches its starting position.

of the man+boat accelerates right.[6] Having acquired motion to the right, the center of mass of man + boat continues by inertia even after the man sits down. Eventually, all slows down due to drag. To recap: the boat started moving left, activating the drag, which caused the center of mass of the whole system to move right—the motion that persisted once the man sat down. Remarkably, the boat will approach its initial position as time goes on. Why this strange coincidence happens is still unclear from the argument just given, but it is explained in the next paragraph.

The **justification** of the remarkable answer is quite simple, but it requires a little calculus. Let $b = b(t)$ denote the position at time t of the boat's center of mass (all is measured in a reference frame of the shore), and, similarly, let $p = p(t)$ be the position of the person (treated as a point mass). The center of mass[7] of the boat–person system is the weighted average of the two positions: $C = C(t) = (mp + Mb)/(m + M)$. Newton's second law (stated on page 172), applied in the direction of the boat's motion, gives

$$(m + M)\ddot{C} = -k\dot{b};$$

here each dot denotes the time derivative. Substituting the expression for C we obtain

$$m\ddot{p} + M\ddot{b} = -k\dot{b}. \qquad (14.1)$$

Let us integrate this relation from $t = 0$ to $t = \infty$. The Fundamental Theorem of Calculus[8] gives $\int_0^\infty \ddot{p}\,dt = \dot{p}(\infty) - \dot{p}(0)$. But $\dot{p}(0) = 0$ because all starts at rest, and

[6] This is a little like a cartoon dog running almost in place: his feet slide backward on the ground (like the boat sliding in water), while the dog's center of mass accelerates forward. Cartoon characters, however, routinely break Newton's laws.

[7] See page 169 for the definition.

[8] See page 184 for the details.

Príklad 4

Solution

a) $r > R$ $\quad \phi_0(r\theta) = -E_0 r\cos\theta + \sum_0^\infty a_n r^{-(n+1)} P_n(\cos\theta)$

$S \leq r \leq R$ $\quad \phi_i(r\theta) = \sum_0^\infty \left(b_n r^n + c_n r^{-(n+1)}\right) P_n$

You can see only terms 0,1 are required to match boundary conditions.

$\phi_i(S\ \theta) = 0 \Rightarrow b_0 + \frac{c_0}{S} = 0 \qquad c_0 = -b_0 S \qquad P_0$

$b_1 S + \frac{c_1}{S^2} = 0 \qquad c_1 = -b_1 S^3 \qquad P_1$

$\underline{\phi_o(R) = \phi_i(R)}$ $\quad b_o(1 - S/R) = a_o/R \quad P_o$

$$① \quad \boxed{b_1 R\left(1 - \frac{S^3}{R^3}\right) = -E_o R + \frac{a_1}{R^2}} \quad P_1$$

$$\underline{-\frac{\partial \phi_o}{\partial r}\Big|_R = -\varepsilon_r \frac{\partial \phi_i}{\partial r}\Big|_R}$$ (D-field normal component continuous)

$$-S/R^2\, b_o \varepsilon_r = a_o/R^2 \quad P_o$$

$$② \quad \boxed{-b_1 \varepsilon_r - 2\frac{S^3}{R^3} b_1 \varepsilon_r = E_o + \frac{2a_1}{R^3}} \quad P_1$$

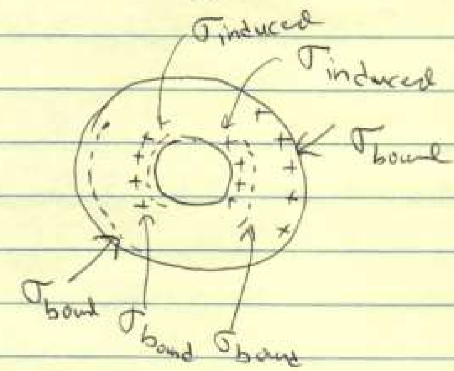
$$\Rightarrow a_o = b_o = 0$$

From ① + ② $$b_1 = \frac{-E_o}{1 - \frac{S^3}{R^3} - \frac{1}{3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}$$

Ans 2 $$\phi_o(r\theta) = -E_o r\cos\theta \left\{1 + \frac{\frac{R^3}{3r^3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}{1 - \frac{S^3}{R^3} - \frac{1}{3}\left[(1-\varepsilon_r) - \frac{S^3}{R^3}(1+2\varepsilon_r)\right]}\right\}$$

A quick check shows that for $S \to 0$ this approaches the dielectric sphere in uniform $\bar{E}$ and for $R \to S$, $\varepsilon_r \to 1$ it approaches the conducting shell in uniform $\vec{E}$.

b)



All charges decreasing towards $\theta = \pi/2$