

Cvičenie 9.12.2025

Príklad 1

PROBLEM: Molecules of an ideal gas have internal energy levels that are equidistant, $E_n = n\varepsilon$, where $n = 0, 1, \dots$ and ε is the level spacing. The degeneracy of n th level is $n + 1$. Find the contribution of these internal states to the energy of the gas of N molecules at temperature T .

Príklad 2

PROBLEM: A point charge $-2q$ is at the origin, $\mathbf{r} = 0$, and two point charges, each $+q$, are at $\mathbf{r} = \pm a\hat{z}$. Consider the limit $a \rightarrow 0$, with $Q = qa^2$ held fixed.

(a) Find the scalar potential $\phi(\mathbf{r})$ in spherical coordinates.

(b) This system of charges is now placed inside a grounded, conducting spherical shell, of radius b (with $b \gg a$). Now find the scalar potential $\phi(\mathbf{r})$ everywhere, both inside and outside of the shell (again, in spherical coordinates).

Príklad 3

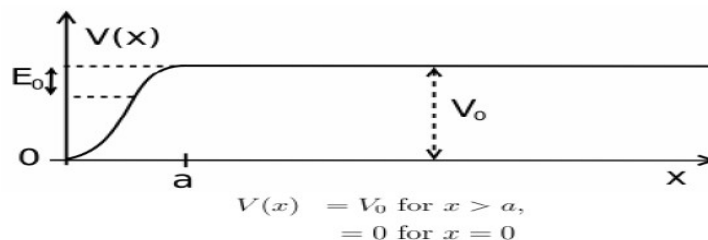
PROBLEM: A particle of mass m is in the ground state of a harmonic oscillator with spring constant $k = m\omega^2$. At $t = 0$, the spring constant changes suddenly to $k' = \lambda^2 m\omega^2$, where λ is a constant. Find the probability that the oscillator remains in its ground state.

PROBLEM: Compute $\langle \psi_0 | x^4 | \psi_0 \rangle$ in the ground state of the one-dimensional harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

Príklad 4

A one-dimensional attractive potential well, $V(x)$, binds a mass m particle to a reflecting wall. The binding energy is $-|E_0|$ relative to $V(x)$ at large distances away.



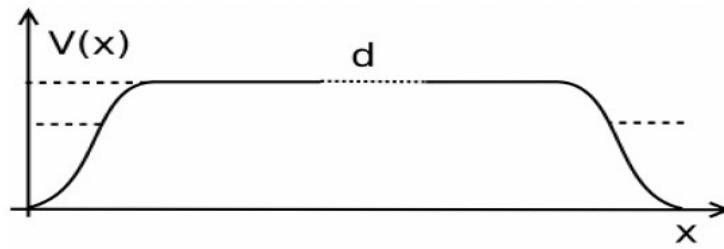
(2)

(3)

For $x \gtrsim a$ the particle wavefunction is $\psi_0 = ke^{-\alpha x}$.

- What is α ?
- Estimate k if the probability for the particle being inside and outside the potential well are comparable.

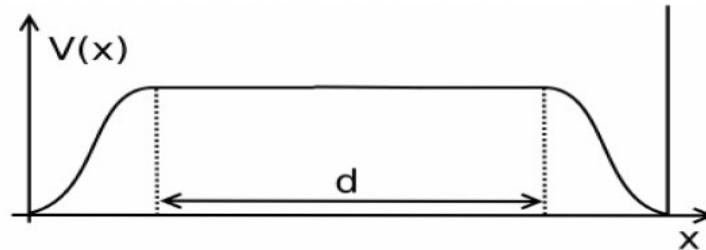
Now suppose that after a long interval d ($d \gg a$), $V(x)$ drops back to its value at $x = 0$.



If at time $t = 0$ the particle is in the state with wavefunction ψ_0 the probability for still finding it near the wall at later times is a diminishing function of time $P(t) \sim e^{-t/\tau}$.

- Estimate τ .

A reflecting wall is now also inserted at a distance $x = a + d + a$ from the first one at $x = 0$ so that the potential and reflector at either end mirror each other.



- What is the new ground state from combining $\psi_0(x)$ and $\psi_0(d - x)$?
- Estimate its tiny energy shift, δE , relative to $-|E_0|$. (Assume that $\psi_0 = ke^{-\alpha x}$ is an adequate approximation for all x .)
- What is the new $P(t)$? How does the time for it to drop to, say, $\frac{1}{2}$ compare to that for c)?