VZOROVÉ RIEŠENIA

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Príklad 1

28. Kľúčovým je uvedomenie si, že nás nezaujíma priebeh pohybu telieska, ale iba počiatočný a koncový stav a tiež fakt, že v gravitačnom poli sa celková energia telesa zachováva. Preto je zákon zachovania energie presne to, čo potrebujeme. Tak si ho pre našu sitáciu zapíšeme:

$$-\kappa\frac{M_1 m}{R_1} - \kappa\frac{M_2 m}{2R_1 + R_2} = \frac{1}{2}mv^2 - \kappa\frac{M_2 m}{R_2} - \kappa\frac{M_1 m}{R_1 + 2R2}$$

Odtiaľto už ľahko dostaneme hľadanú rýchlosť telieska:

$$v = 2\sqrt{\kappa\left(M_2\frac{R_1}{R_2(2R_1 + R_2)} - M_1\frac{R_2}{R_1(R_1 + 2R_2)}\right)}$$

Príklad 2

ako predtým, pomôcť môžu poznámky https://davinci.fmph.uniba.sk/~tekel1/docs/zleriesenia.pdf .

Príklad 3

$$E = 0 = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy$$

$$y = -ax^n \;\Rightarrow\; \dot{y} = -nax^{n-1}\dot{x}$$

$$\Rightarrow\; \dot{x}^2 = \frac{2gax^n}{1 + n^2 a^2 x^{2(n-1)}}$$

Force of constraint in the x direction is $Q_x = m\ddot{x} = 0$ if particle leaves the surface.

$$2\not{\dot{x}}\,\ddot{x} = \frac{\partial}{\partial t}\left(\frac{2gax^n}{1 + n^2 a^2 x^{2(n-1)}}\right) = \not{\dot{x}}\frac{\partial}{\partial x}\left(\frac{2gax^n}{1 + n^2 a^2 x^{(2n-2)}}\right) = 0$$

$$\Rightarrow\; \frac{nx^{n-1}}{1 + n^2 a^2 x^{2(n-1)}} - \frac{2(n-1)n^2 a^2 x^{3n-3}}{(1 + n^2 a^2 x^{2n-2})^2} = 0$$

$$nx^{n-1} + n^3 a^2 x^{3n-3} - 2(n-1)n^2 a^2 x^{3n-3} = 0$$

$$\Rightarrow\; a^2 x^{2n-2} = \frac{1}{n(n-2)}. \text{ Real finite solution only for } n > 2.$$

Príklad 4

a) The effective potential per unit mass is

$$V_{\text{eff}}(r) = \epsilon - \frac{\dot{r}^2}{2} = \frac{\ell^2}{2r^2} - \frac{K^2}{r^4},$$

where ϵ is the energy per unit mass and ℓ is the angular momentum per unit mass. The circular orbit has $dV_{\text{eff}}/dr = 0$ or $r_{\text{circ}} = 2K/\ell$. Hence the period is $P = 2\pi r^2/\ell = \pi r^3/K$. V_{eff} has a maximum at r_{circ} and so the equilibrium is unstable.

b) A spaceship with $\epsilon < V_{\text{circ}} = \ell^4/16K^2$ will be repelled by the effective potential and so capture will be avoided if

$$\frac{1}{2}v^2 = \epsilon < V_{\text{circ}} = \frac{\ell^4}{16K^2} = \frac{v^4 b^4}{16K^2}$$
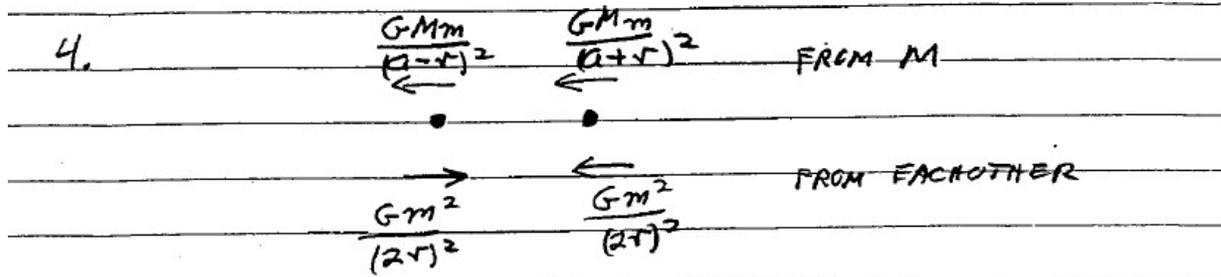
where b is the impact parameter. Hence

$$b > \left(\frac{8K^2}{v^2}\right)^{1/4}$$

to avoid capture.

c) Just treat the object as having a scattering cross-section of b. Then the mass capture rate is $\pi b^2 \times \rho v = 2\sqrt{2}\pi K \rho$.

Príklad 5

4.

$$\underbrace{\frac{GMm}{(a-r)^2}}_{\leftarrow} \quad \underbrace{\frac{GMm}{(a+r)^2}}_{\leftarrow} \quad \text{FROM } M$$

$$\bullet \qquad \bullet$$

$$\underbrace{\frac{Gm^2}{(2r)^2}}_{\rightarrow} \quad \underbrace{\frac{Gm^2}{(2r)^2}}_{\leftarrow} \quad \text{FROM EACHOTHER}$$

FOR STABILITY, FORCE TO THE LEFT ON THE LEFT OBJECT MUST BE LESS THAN THE FORCE TO THE LEFT ON THE RIGHT OBJECT

$$\frac{GMm}{(a-r)^2} - \frac{Gm^2}{4r^2} < \frac{GMm}{(a+r)^2} + \frac{GMm\,m^2}{4r^2}$$

$$\frac{M}{(a^2-r^2)^2}(a+r)^2 - \frac{M}{(a^2-r^2)^2}(a-r)^2 < \frac{m}{2r^2}$$

$$4ar\frac{M}{a^4} < \frac{m}{2r^2}$$

$$\frac{8Mr^3}{a^3} < m = \frac{4}{3}\pi r^3 \rho$$

$$\frac{6}{\pi}\frac{M}{a^3} < \rho$$